CEIS TOR VERGATA Research Papers

Università di Roma Tor Vergata

ISSN 2610-931X

CEIS Tor Vergata

RESEARCH PAPER SERIES

Vol. 23, Issue 5, No. 604 – June 2025

New Keynesian Models, Puzzles and Omitted Effects

Francesco Morelli

New Keynesian Models, Puzzles and Omitted Effects

Francesco Morelli *

June 19, 2025

Abstract

The modern macroeconomic debate grapples with explaining complex phenomena, often giving rise to theoretical and empirical puzzles. In response, New Keynesian models have evolved, incorporating frictions and agent heterogeneity, leading to the development of TANK and HANK frameworks. However, increasing model complexity does not always clarify underlying causes. This work introduces a New Keynesian model with Omitted Effects, incorporating previously unaccounted-for variables into agents' objective functions. Similar to Omitted Variable Bias in econometrics, neglecting key variables in a general equilibrium setting leads to model misspecification and puzzling anomalies. The study follows both theoretical and empirical approaches: first, demonstrating how Omitted Effects address the Forward Guidance Puzzle and the consumption response to government spending; second, using an algorithm to identify omitted variables that improve model fit to US data. Empirical findings highlight the crucial role of real wages, inflation, and income Omitted Effects in resolving key macroeconomic inconsistencies. This framework enhances DSGE models' flexibility, providing a systematic tool to refine economic modeling and deepen the understanding of macroeconomic dynamics.

Keywords: DSGE, Omitted Effects, Forward Guidance Puzzle, Government Spending, Identification, General Equilibrium. Augmented Objective Function.
JEL Codes: E10, E20, E21, E24, E27,E30, E31, E37, E52, E62.

*Link Campus University, Rome. Italy. E-mail: f.morelli@unilink.it

1 Introduction

The modern macroeconomic debate encompasses numerous themes of fundamental importance, both on the normative and the positive side. Economists strive to find coherent and univocal explanations for complex phenomena, which often appear contradictory. This ongoing quest has led to the emergence of theoretical and empirical "puzzles" and, consequently, the development of new models aimed at resolving those. Initially, New Keynesian models were enhanced by introducing frictions on both the demand and supply sides. This evolution led to the possibility of agent heterogeneity, giving rise to Two-Agent New Keynesian (TANK) models, followed by Heterogeneous Agents New Keynesian (HANK) models, which have since become prominent in the macroeconomic landscape[1].

However, the proliferation of models and hypotheses intended to explain complex phenomena can lead to increased complexity without necessarily clarifying consistently their deep causes. The analytical tool I propose in this work involves the definition and the introduction of Omitted Effects into a baseline model, by incorporating non-choiche variables within one of the objective functions being maximized. I define this model as a New Keynesian model with Omitted Effects. Similar to what occurs in an econometric context, where the absence of relevant variables in an OLS model can lead to the emergence of Omitted Variable Bias (OVB), in a general equilibrium setting, the absence of key variables in critical equations can result in model misspecification and the emergence of inconsistencies and related puzzles. The issue we are addressing is, hence, an identification problem in general equilibrium. The motivations for introducing this new framework are manifold. First, the aim is to enrich pre-existing models to interpret and quantify macroeconomic phenomena more precisely. The second goal is to make a certain model more flexible, allowing for a better fit with the data on the one hand and addressing macroeconomic puzzles with greater adaptability on the other. Finally, the purpose is to provide researchers with an intermediate analytical tool, employing Omitted Effects to identify the "weak points" of an existing model, thereby enabling its improvement through a consistent and adequate microfoundation.

This work is structured to address the themes of puzzles and model misspecification through two complementary approaches: the theoretical and the empirical one. After introducing the framework of Omitted Effects in DSGE models in Chapter 3, I adopt a theoretical approach in Chapter 4 by incorporating two Omitted Effects, the ones related to income and the inflation rate into a vanilla new Keynesian model. This allows me to address two significant puzzles: the Forward Guidance one in monetary policy and the one related to the consumption response to a government spending shock.

In Chapter 5, I follow an empirical approach, employing an algorithm to select Omitted Effects that improve the fit of a RANK model to US data. This process identifies the optimal combination (given a certain information criterion) of admissible effects. The empirical results confirm an improvement in the model's fit to the data when omitted variables are introduced within the household's utility function. Three variables were identified as having a significant unaccounted-for impact in the RANK model, concerning the household side: real wages, inflation, and income. The model exhibited several interesting characteristics of the impulse response functions. About inflation, an exogenous shock generates a positive effect on consumption and income, while simultaneously causing a significant drop in real wages. A shock to the consumption component of public spending has instead an almost negligible impact on household consumption, as well as on inflation and the policy interest rate. This outcome contradicts both the predictions of the RANK

[1] See Kaplan and Violante 2018 for the first quantitative version of HANK, and Florin Ovidiu Bilbiie 2018 for an analytical version.

model, which anticipates a significant reduction in consumption and those of the TANK model, which predicts a substantial increase in household consumption.

Paper Overview. The paper is structured as follows. Chapter 2 presents several contributions related to this work. Chapter 3 introduces the framework of the Omitted Effects, focusing on the definition of the novel elements introduced and the various technical aspects necessary for understanding and applying the framework. In Chapter 5, I present an example of a RANK model with two omitted effects—those of inflation and the nominal interest rate—and analyze the conditions under which the two puzzles can be resolved. Chapter 5 proposes an application of the omitted effects selection algorithm in a RANK model through the empirical estimation of the omitted effects themselves. Section 6 concludes the paper.

2 Related Literature

This work is related to various branches of literature. Three main macro-categories have been identified: general content on DSGE models, literature on macroeconomic puzzles, and studies concerning deviations from the representative agent and perfect rationality.

Since this work is both generalist and methodological in nature regarding Dynamic Stochastic General Equilibrium (DSGE) models, it is crucial to consider the relevant literature. This includes both the foundational pillars of DSGE models and the studies that have offered significant critiques. Starting from Kydland and Prescott 1982 real business cycle model, through more recent contributions such as those by L. J. Christiano, Eichenbaum, and Evans 2005, as well as Smets and Wouters 2007, the evolution from the real business cycle school to the new Keynesian models widely used in academia and economic policy has involved the introduction of progressively more assumptions. These assumptions aim to align the models with empirical data and to generate scenarios useful for policy decision-making.

However, DSGE models faced, and continue to face, severe criticism from various perspectives, including their failure to predict the 2007–2008 financial crisis, the arbitrariness of their assumptions, lack of realism, excessive simplification, and neglect of non-linearities. Some of these them can be found in the works of Stiglitz 2018 and Korinek 2017.

The second macro-category of literature concerns macroeconomic puzzles. These puzzles manifest as inconsistencies between pre-existing theoretical models and certain aspects of empirical evidence or stylized economic facts. Such inconsistencies, and the emergence of the puzzles, represent a significant challenge for any theoretical model, as they undermine confidence in the model's ability to predict economic behaviour and serve as a useful tool for economic policy decision-making.

I reference these puzzles for two reasons. First, their existence sparks debate around the identification of DSGE models, which, in turn, motivates the introduction of Omitted Effects. Second, throughout this work, I will frequently refer to them.

Among the most notable puzzles in macroeconomic debate, this study focuses on two: the government spending puzzle and the forward guidance puzzle.

The first one concerns a highly relevant issue in macroeconomics and economic policy: the magnitude of the expansionary effect of public spending on the economy. From the Keynesian tradition onward, public spending policies have been central to both economic stabilization strategies and resource redistribution efforts. However, whether this impact on private consumption is positive or negative has long been a subject of heated debate.

On one side, the neoclassical school, embodied in the real business cycle framework, finds empirical evidence for the crowding out of consumption following an increase in public spending. On the other side, new Keynesian empirical evidence, often based on structural vector autoregression (SVAR) approaches, suggests the opposite effect.

From a theoretical perspective, certain macroeconomic structures and models can represent both phenomena. In Baxter and King 1993, for instance, public spending positively affects consumption only when it leads to increased investment; when it primarily drives consumption increases, the effect is negative[2].

In contrast, theoretical models predicting a rise in consumption, such as those by Galí, López-Salido, and Vallés 2007 and Corsetti, Meier, and Müller 2012, rely on different mechanisms. Galí emphasizes the hypothesis of borrowing-constrained agents, while Corsetti highlights the importance of fiscal policy rules. A definitive answer remains elusive. For a comprehensive survey on the subject, see Ramey and Zubairy 2018[3]. In this work, I demonstrate how Omitted Effects can help to cure the government spending puzzle. By initially introducing the effects of inflation and income into the household utility function, I studied the conditions under which private consumption responds positively to public consumption. From an empirical perspective, using the Omitted Effects selection algorithm proposed in this study, I show that incorporating them leads to a model where the impact of public spending on consumption approaches zero. This result represents a middle ground between the two contrasting views and is certainly at odds with extreme positions regarding the role of public spending in income stabilization.

The second issue addressed is the forward guidance puzzle. When policy interest rates hit the zero lower bound, a growing body of literature focused on potential strategies for escaping the liquidity trap[4].

In their work, Eggertsson et al. 2003 propose that the central bank commits to setting not only current interest rates but also future rates, to influence agents' expectations about future rates today. According to the authors, this mere announcement of future rates is sufficient to prevent a deflationary spiral. However, as shown in numerous studies, including those by Carlstrom, Fuerst, and Paustian 2015, as well as Del Negro, Giannoni, Patterson, et al. 2012, the predictions of new Keynesian models regarding the impact of forward guidance are overly optimistic in terms of both output and inflation responses. These studies demonstrate that an exogenous change in nominal interest rates five years into the future has an impact 18 times greater than the same reduction made today. This result does not align with empirical observations, casting doubt on the ability of workhorse models, such as those by Smets and Wouters 2007, to adapt to and predict scenarios like the zero lower bound and the impact of forward guidance. Numerous authors, including Svensson

[2] This occurs through the so-called RANK channel, where the negative effect on consumption arises from an increase in the real interest rate, which follows a rise in inflation and corresponding monetary policy adjustments.

[3] The debate on government spending multipliers has been central since the 1980s, with seminal works like Baxter and King 1993 Neo-Classical model suggesting variations in multipliers depending on financing channels and predicting "crowd-out" effects due to rational agent behaviour. Empirical studies, such as Ramey and Shapiro 1998, supported these predictions using military events as exogenous shocks, showing a positive but subunit multiplier. New Keynesian models, incorporating market imperfections, indicated a positive multiplier, particularly on wages and employment. State-dependent analysis revealed countercyclical responses, while Auerbach and Gorodnichenko 2012 noted multiplier variations with the economic cycle. The Zero Lower Bound condition highlighted by L. Christiano, Eichenbaum, and Rebelo 2011 and Eggertsson and Woodford 2004 suggested multipliers could exceed one. Alternative structures like Corsetti, Meier, and Müller 2012 "spending reversal" and studies by Forni and Gambetti 2010 and Bernardini and Peersman 2018 explored the influence of public and private debt on fiscal policy. Overall, new theoretical and empirical insights continue to refine our understanding of spending multipliers, emphasizing the importance of economic context.

[4] The latter, theoretically capable of generating a deflationary spiral as first described by Eggertsson et al. 2003, and Eggertsson and Woodford 2003, motivated many economists to seek solutions to this critical problem.

2014, McKay, Nakamura, and Steinsson 2016, have proposed models capable of explaining the ineffectiveness of forward guidance. These solutions revolve around Euler discounting, which implies that agents' expectations are discounted for various reasons[5]. In this work, I will demonstrate how Omitted Effects serve as a valuable tool for addressing the forward guidance puzzle. In Chapter 4, I will illustrate how inflation and its Omitted Effect in the RANK model play a pivotal role in resolving the forward guidance puzzle, as they can generate Euler discounting even in the presence of an interest rate peg.

This work is also connected to three branches of macroeconomic literature concerning TANK models, HANK models, and models of Cognitive Discounting. Throughout the study, we will refer to these frameworks on multiple occasions. First, the assumptions introduced by these models represent a cornerstone of modern macroeconomic models and are employed specifically to address the puzzles discussed in this work.

The earliest formulation of the TANK model is attributed to Galí, López-Salido, and Vallés 2007. In this model, the authors introduce a basic form of household heterogeneity by assuming that some agents lack access to financial markets and can only consume their disposable income. This assumption effectively resolves the government spending puzzle and aligns the theory with the empirical findings of O. Blanchard and Perotti 2002.

More recent variations on this theme are due to Florin O Bilbiie 2008 and Florin O Bilbiie and Straub 2012, within a strand of research known as Limited Asset Market Participation (LAMP). This approach yields similar results but through a different transmission mechanism, employing the Inverted Aggregate Demand Logic (a reversal of the slope of aggregate demand) to explain phenomena and address puzzles. The empirical foundation of these two linked frameworks is the excess sensitivity theory of consumption toward aggregate income [6].

To address the resolution of the forward guidance puzzle and the simultaneous resolution of both puzzles, we must take an additional step and turn to HANK models that incorporate idiosyncratic risk. An analytical version of the quantitative HANK model proposed by Kaplan, Moll, and Violante 2018 can be found in Florin Ovidiu Bilbiie 2018, where agent heterogeneity and idiosyncratic risk, combined with procyclical risk, are capable of resolving both the government spending puzzle and the forward guidance puzzle simultaneously. Finally, the framework introduced by Gabaix 2020, through the incorporation of limitations to perfect rationality, yields similar implications and offers a comparable resolution to the puzzles under examination.

3 Methodology

The aim of this work is to provide a methodology capable of flexibly address theoretical issues experienced in macroeconomic research. The emergence of puzzles, defined as the inability of

[5]For instance, the presence of incomplete markets and idiosyncratic risk can generate Euler discounting, as can precautionary savings or money-in-the-utility frameworks, yielding similar results. Likewise, inFlorin Ovidiu Bilbiie 2018, idiosyncratic risk and agent heterogeneity under certain conditions lead to Euler discounting. Finally, limitations to perfect rationality, such as cognitive discounting as described by Gabaix 2020 and Gabaix and Laibson 2017, significantly diminish the effectiveness of forward guidance.

[6]Studies on aggregate consumption date back to Keynes and Friedman, with Keynesians linking consumption to current disposable income and the "Ricardian" view tying it to permanent income. Hall 1978 Life-Cycle hypothesis suggested consumption follows a random walk, while Flavin 1984 showed its sensitivity to current income. Dornbusch and Fischer 1993 highlighted the impact of liquidity constraints, and various authors considered household heterogeneity in borrowing capabilities, leading to non-Ricardian behaviour. J. Y. Campbell and Mankiw 1989 developed a model combining both behaviours, finding that consumption is closely tied to current income, with no evidence of Consumption Smoothing.

a model or an entire framework to theoretically account for an empirical fact or to consistently explain phenomena, requires a methodological effort. These inconsistencies highlight potential flaws in the model and its ability to accurately represent economic events[7]. Similar to what occurs in econometrics, where the absence of relevant variables can lead to the emergence of omitted variable biases, in general equilibrium, the absence of certain variables in some of the fundamental equations can lead to misspecification. This affects the general equilibrium, with consequences that propagate to the model's ability to predict and assess the impact of macroeconomic shocks. The methodology proposed involves the introduction of a framework designed to assist researchers in dealing with puzzles and tackle misspecification within a DSGE environment.

The analytical tool entails the modification of one or more objective functions of a baseline model, to explicitly account for variables whose omission may be a potential source of misspecification. Rather than referring to omitted variable bias in the econometric sense, in the general equilibrium framework, I define this bias as the Omitted Effect.

Besides tackling macroeconomic puzzles, the model with Omitted Effects, which constitutes an isomorphic extension of the baseline model, serves other purposes. First, it enhances the baseline model to better represent economic facts. The second objective is to provide greater analytical flexibility in theoretical settings and empirical analysis. Finally, it is intended to serve as a valuable ally for economists and as an intermediate research tool, capable of identifying a model's weaknesses, which can then be addressed.

The methodology presented in this chapter includes the definitions of three key elements throughout the chapter: the Omitted Effect Objective Function, which we will also refer to as the Augmented Objective Function, the definition of the Macroeconomic Puzzle, and that of the Omitted Effect.

Let me introduce the primary analytical tool: the Omitted Effect Objective Function.

Definition 1. *Let $\Omega_t = \mathcal{U}(\mathcal{C}_t)$ be the generic objective function associated with the baseline model, where $\mathcal{U}_t(\cdot)$ is a continuous, non-linear function. $\mathcal{C}$ is a vector of choice variables indexed at time t, with $\mathcal{C} \in \mathcal{A}$. $\mathcal{A}$ represents the set of endogenous and exogenous variables included within the baseline model.*

Let

$$\Omega_t^{OE} = \mathcal{U}^{OE}(\mathcal{C}_t, \mathcal{X}_t) \tag{1}$$

denote the objective function of the model with Omitted Effects (the Augmented model), where $\mathcal{U}^{OE}$ is another continuous, non-linear function. $\mathcal{X}$ represents a vector of non-choice variables belonging to $\mathcal{A}^{OE}$, where $\mathcal{A}^{OE} \equiv \mathcal{A}$. Ω_t^{OE} is defined as the ***Omitted Effect Objective Function*** *or the* ***Augmented Objective Function****.*

The Augmented objective function represents an extension of the baseline one. Furthermore, the two models share the same number and type of variables.

Assumption 1. *To convey its properties in general equilibrium, the Omitted Effect Objective Function must be non-separable between $\mathcal{C}$ and $\mathcal{X}$. Specifically, it is necessary for $\mathcal{X}$ to be non-separable with respect to at least one variable contained in $\mathcal{C}$. This occurs when:*

$$\frac{\partial \Omega_t^{OE}}{\partial \mathcal{C}_t} = g(\mathcal{C}_t, \mathcal{X}_t). \tag{2}$$

[7] A puzzle is an inconsistency between empirical evidence the inability a model to represent it. This assumes that the empirical evidence is accurate or at least well-informed.

Equation 2 implies ***Omitted Effects Non-Separability****.*

This assumption is necessary to ensure that the properties of the objective function are inherited by the rest of the model. In general equilibrium, where the model is derived from micro foundation, for a characteristic to be captured by the equations it is essential that such characteristics are inextricably linked to at least one choice variable—i.e., a variable through which the optimization process is carried out.

Example 1. *An objective function that satisfies the criterion outlined in Assumption 1 is the one with Cobb-Douglas Omitted Effects:*

$$\Omega_t^{OE} = \mathcal{U}(\mathcal{C}_t) \prod_{j=1}^{m} \mathcal{X}_{j,t}^{\gamma_j}, \tag{3}$$

where m is the number of non-choice variables included in non-separable form with the choice variables, or at least one of them. γ^j is the elasticity of Ω_t^{OE} whit respect to the j-th non-choice variable.

Consider as an example the instantaneous utility function of the representative agent in a baseline model, where utility positively depends on consumption and negatively on hours worked, $U_t = \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{N_t^{1+\eta}}{1+\eta}$, where σ represents the coefficient of Relative Risk Aversion, η is the inverse of the Frish Labour Supply Elasticity and χ is a scale parameter. A Cobb-Douglas Omitted Effect Objective Function relative to U_t is:

$$U_t^{OE} = \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{N_t^{1+\eta}}{1+\eta} \right) \prod_{j=1}^{m} \mathcal{X}_{j,t}^{\gamma_j}. \tag{4}$$

In eq. (4) The m non-choice variables are non-separable with both household consumption and hours worked. Assumption 1 is met as long as the partial derivatives of U_t^{OE} with respect to both C_t and N_t (choice variables), are functions of $\mathcal{X}_t$:

$$\frac{\partial U_t^{OE}}{\partial C_t} = C_t^{-\sigma} \prod_{j=1}^{m} \mathcal{X}_{j,t}^{\gamma_j},$$

$$\frac{\partial U_t^{OE}}{\partial N_t} = -\chi N_t^{\eta} \prod_{j=1}^{m} \mathcal{X}_{j,t}^{\gamma_j}.$$

Lemma 3.1 (Isomorphism of Objective Functions)**.** *Let $\Omega_t = \mathcal{U}(\mathcal{C}_t)$ be the objective function of the baseline model, and let $\Omega_t^{OE} = \mathcal{U}^{OE}(\mathcal{C}_t, \mathcal{X}_t)$ represent the augmented objective function with Omitted Effects. The two objective functions, Ω_t and Ω_t^{OE}, are isomorphic in the sense that there exists a transformation $\mathcal{T}$ that preserves their structural properties. Specifically, the following holds:*

$$\Omega_t^{OE} = \mathcal{T}(\Omega_t) \quad \textit{with} \quad \mathcal{T} : \mathbb{R}^n \to \mathbb{R}^n \tag{5}$$

where $\mathcal{T}$ is an isomorphic transformation that relates the baseline objective function Ω_t to the augmented objective function Ω_t^{OE}. The transformation preserves the relationships between the choice variables $\mathcal{C}_t$ and the optimality conditions of the model, despite the introduction of non-choice variables $\mathcal{X}_t$. Therefore, the two functions have the same fundamental structure, and the introduction of $\mathcal{X}_t$ in Ω_t^{OE} does not alter the core optimization process that involves the choice variables.

Thus, the augmented objective function Ω_t^{OE} can be considered an extension of Ω_t that preserves the same optimization properties and relationships, maintaining isomorphism between the two functions.

Now we can move from the objective function to the entire model. I follow Fernández-Villaverde, Rubio-Ramírez, and Schorfheide 2016 for the nomenclature.

Let's start from the baseline model equilibrium solution and its steady state:

$$\mathbb{E}_t \mathcal{H}(Y_t, Y_{t+1}, X_t, X_{t+1}) = 0 \tag{6}$$

$$\mathcal{H}(\bar{Y}, \bar{Y}, \bar{X}, \bar{X}) = 0 \tag{7}$$

where $\mathbf{Y}$ is an $n_y \times 1$ vector of controls, $\mathbf{X}$ is an $n_x \times 1$ vector of states, and $n = n_x + n_y$. The operator $\mathcal{H} : \mathbb{R}^{n_y} \times \mathbb{R}^{n_y} \times \mathbb{R}^{n_x} \times \mathbb{R}^{n_x} \to \mathbb{R}^{n_y}$ stacks all the equilibrium conditions, some of which will contain expectational terms, while others will not. Equation (7) represents the steady state solution of the baseline model. Consider now the first-order perturbation of the model in the vicinity of the steady state:

$$\begin{aligned} g(x;\theta)y &= g_x(x;0)(x-\bar{x}) + g(x;0), \\ h(x;\theta)x &= h_x(x;0)(x-\bar{x}) + h(x;0), \end{aligned}$$

The two equations represent the first-order Taylor expansions of the functions $g(x;\theta)$ and $h(x;\theta)$ around the steady-state point $\bar{x}$, where θ denotes the parameter vector.

The solution of the model is given by a set of decision rules for the control variables:

$$y_t = g(x_t;\theta), \tag{8}$$

and for the state variables:

$$x_{t+1} = h(x_t;\theta) + \sigma\eta\varepsilon_{t+1}, \tag{9}$$

where ε represents the exogenous shocks affecting the system.

I now focus on the equation governing the state variables, as the control variables can subsequently be uniquely expressed as a function of x. The Data Generating Process (DGP) of the log-linearized model is:

$$x_t = \Gamma(\theta)x_{t+1} + \zeta(\theta)\varepsilon_t. \tag{10}$$

Γ and ζ are two functions of the parameter set θ. To proceed with the analysis and introduce the concept of macroeconomic puzzle, I frame the discussion in terms of Impulse Response Functions. An **Impulse Response Function (IRF)** describes the dynamic response of an endogenous variable in a system to a one-time, unitary shock to an exogenous variable, while holding all other shocks constant. I first solve Equation (10) with respect to the shock itself through the method of undetermined coefficients. Assume $x_t = \xi\varepsilon_t$. Substituting and solving for ε_t, given that the exogenous shock follows an AR(1) process $\varepsilon_t = \rho\varepsilon_{t-1} + \nu_t$, where ν_t is normally distributed with mean zero and constant standard deviation σ_ν. ρ is the autoregressive coefficient of the process for ε_t. The solution entails the characteristic equation:

$$(\xi - \xi\Gamma(\theta) - \xi\zeta(\theta)) = 0. \tag{11}$$

That solving for $\xi = \frac{\zeta(\theta)}{1-\Gamma(\theta)\rho}$ brings to:

$$x_t = \mathcal{M}(\theta)\varepsilon_t, \tag{12}$$

where $\mathcal{M}(\theta) = \left(\frac{\zeta(\theta)}{1-\Gamma(\theta)\rho}\right)$.

Definition 2. *Let $\mathcal{F}\varepsilon$ denote the matrix of impulse response functions associated with the hypothesis contained in $\mathcal{F}$. $\mathcal{M}(\theta)\varepsilon$ represents the matrix of impulse response functions of the baseline model. Let Ω_θ denote the admissible parameter space. A **macroeconomic puzzle** is defined as the **non-existence** of at least one combination of $\theta \in \omega_\theta$ such that $\mathcal{M}(\theta)\varepsilon = \mathcal{F}\varepsilon$.*

Proposition 1. *A solution to the macroeconomic puzzle exists in the form of a model with Omitted Effects, isomorphic to the baseline model, if there exists at least one θ_{OE} such that $\mathcal{M}(\theta_{OE}) = \mathcal{F}$. Where θ_{OE} is the set of coefficients of the model with Omitted Effects that includes θ and the additional coefficients, one for each Omitted Effect.*

In other words, $\mathcal{F}$ represents an external benchmark, such as empirical evidence or a stylized fact. The objective here is to assess the ability of the baseline model to represent the thesis encapsulated in $\mathcal{F}$. If there exists a combination of θ such that the baseline model can replicate the case $\mathcal{F}$, then no puzzle is present. Conversely, if this is not the case, the puzzle exists, and a solution with Omitted Effects is possible under the conditions described above. The conditions pertaining to the model with Omitted Effects, as outlined thus far, must hold.

If, however, there does not exist even a single combination of θ_{OE} capable of representing $\mathcal{F}$, this suggests that no solution with Omitted Effects to the macroeconomic puzzle exists, and the framework should be modified. However, in the opposite case, the solution with Omitted Effects provides useful information about the potential origin of the puzzle. Furthermore, reference has been made to admissible values for the set of parameters θ. In fact, for a puzzle to be solvable through Omitted Effects, while the coefficients on the latter have a free domain, by assuming an uninformative prior, the deep parameters of the baseline model can be constrained by the underlying microeconomic theory. Consider, for example, the Calvo coefficient or other parameters whose domain is restricted to a specific range: in this case, it is necessary that the coefficients solving the puzzle fall within their respective intervals.

All the elements are in place to introduce the concept of Omitted Effect.

Definition 3. *Let $\mathcal{M}_{OE}$ be the matrix of coefficients of the impulse response functions of the model with Omitted Effects, isomorphic to the baseline model, and let $\mathcal{M}$ be the same matrix referring to the baseline model. Where θ_{OE} is the set of coefficients of the model with Omitted Effects, and θ is the set of coefficients of the baseline model. θ_{OE} consists of n coefficients shared with θ and m coefficients γ_j related to the omitted effects. It is possible to identify three categories of **Omitted Effects**:*

- *The Overall Omitted Effect*

$$\mathcal{O} = \mathcal{M}_{OE} - \mathcal{M} \tag{13}$$

- *The j-th Omitted Effect*

$$\mathcal{O}_j = \mathcal{M}_{OE}|_{j\neq 0, i=0 \forall i\neq j} - \mathcal{M} \tag{14}$$

- *The ij-th Interaction Omitted Effect*

$$\mathcal{O}_{ij} = \mathcal{M}_{OE} - \sum_j^m \mathcal{O}_j \tag{15}$$

The Overall Omitted Effect consists of the difference in the shock coefficient between the model with m Omitted Effects and the isomorphic model without Omitted Effects, that is, where $\gamma_j = 0$ for each $j \in m$.

The j-th Omitted Effect is the difference in the shock coefficient between the model with only the Omitted Effect of the j-th variable belonging to m, and the model without any Omitted Effects.

The ji-th Interaction Omitted Effect is the omitted effect due to the interaction between the two non-choice variables, j and i, belonging to m. It is defined residually with respect to the overall effect and the effect of each individual non-choice variable. There are as many interactions as there are partitions of the omitted effects in m, excluding the omitted effects of the individual variables.

The construct of Omitted Effects pertains to two key elements that characterize it. First is the baseline model to which they refer. Indeed, two different baseline models, unless they are isomorphic to one another, cannot be compared in terms of omitted effects. Second, the functional form. Starting from the same baseline model, there is no single way to introduce omitted effects; rather, multiple approaches exist, each corresponding to a specific combination of resulting effects. In fact, the objective function that can be modified may refer to household utility, the production function, or other key equations, such as monetary or fiscal policy rules, or even various combinations of these. Furthermore, even within the utility function, there are multiple ways to apply these modifications. In the following paragraphs, I will illustrate some types of objective functions and how they can be expanded through the introduction of Omitted Effects, addressing separately the household and firm perspectives.

Hoseholds.

Regarding households, I will modify the utility function in four distinct ways, each corresponding to a different microfoundation of the Omitted Effects and leading to different propagation mechanisms.

Type 1:

$$U_t^{OOC} = \sum_{k=0}^{\infty} \beta^k \left(\frac{C_{t+k}^{1-\sigma}}{1-\sigma} \prod_{j=1}^{m} \mathbf{X_{j,t+k}}^{\gamma^j} - \frac{N_{t+k}^{1+\eta}}{1+\eta} \right) \tag{16}$$

Type 2:

$$U_t^{OOC} = \sum_{k=0}^{\infty} \beta^k \left(\frac{C_{t+k}^{1-\sigma}}{1-\sigma} - \frac{N_{t+k}^{1+\eta}}{1+\eta} \right) \prod_{j=1}^{m} \mathbf{X_{j,t+k}}^{\gamma^j} \tag{17}$$

Type 3:

$$U_t^{OOC} = \sum_{k=0}^{\infty} \beta^k \left(\frac{C_{t+k}^{1-\sigma}}{1-\sigma} - \frac{N_{t+k}^{1+\eta}}{1+\eta} \prod_{j=1}^{m} \mathbf{X_{j,t+k}}^{\gamma^j} \right) \tag{18}$$

Type 4:

$$U_t^{OOC} = \sum_{k=0}^{\infty} \beta^k \left(\frac{C_{t+k}^{1-\sigma}}{1-\sigma} \prod_{j=1}^{m} \mathbf{X_{j,t+k}}^{\gamma^j} - \frac{N_{t+k}^{1+\eta}}{1+\eta} \prod_{i=1}^{n} \mathbf{X_{i,t+k}}^{\delta^i} \right) \tag{19}$$

The four Augmented Objective Functions represent different approaches to the implementation of Omitted Effects into a baseline model.

In the first case, eq. (16), the Cobb-Douglas combination of non-choice variables is non-separable only with respect to the consumption factor. In eq. (17), the new component is non-separable with respect to both consumption and labour. The third type, in eq. (18), is non-separable only to labour. Finally, in the fourth case in eq. (19), there are two sets of Omitted Effects, M and N, which are independent and separately influence household utility. These proposed typologies are not exhaustive, and I encourage readers to formulate their own[8].

Concerning the economic implications of the different typologies, each one will lead to alternative consequences and transmission channels.

In the first case, following the optimization process, both the consumption and the wage equations will be influenced by these variations. However, the building blocks of wage determination are the same as those influencing consumption, as the set of non-choice variables is identical. Thus, the Omitted Effects will impact both aggregate demand and labour supply, also playing a role on the supply side.

In the second typology, however, the framework is quite different. The non-control variables are linked in a non-separable form with both components. When solving the maximization problem, however, the wage equation returns to that of the baseline model. In this case, the Omitted Effects are therefore limited to aggregate demand, displaying no impact on the supply side.

In the last cases, we encounter distinct scenarios: in case 3, the Omitted Effects are limited to labour supply and thus only affect the wage equation, while in case 4, the they operate independently on both aggregate demand and labour, resulting in two separate transmission channels.

Firms.

As an objective function, we can modify the production function of firms either directly or through the profit function.

$$Y_t = A_t N_t^{\alpha} \prod_{i=1}^{n} \mathbf{X_{i,t}}^{\delta i} \tag{20}$$

What is implicitly assumed is that production is a complex process in which, beyond labour, other factors can influence output[9].

Public Authorities.

Finally, monetary and fiscal policy rules can also be influenced by the presence of Omitted Effects. In this case, we enter the domain of normative economics. While identifying relevant Omitted Effects in the rules governing economic dynamics—such as the Taylor rule in monetary policy or public budget policies—can be a valuable endeavour, we leave the consideration of such possibilities to the reader and to future studies.

4 A New Keynesian Model with Omitted Effects

In this section, I introduce a baseline New Keynesian model (RANK) augmented with Omitted Effects. The purpose of this chapter is multifaceted. On the one hand, it serves an illustrative

[8]For example, if a framework with Money-in-Utility is employed, the monetary component would add another layer for Omitted Effects to interact with, exponentially increasing the partitions.

[9]If this may seem unorthodox, consider, for example, that technology (the Solow residual), defined by Abramowitz as "the measure of our ignorance," is identified residually with respect to labour (and capital). It is not necessarily the case that a portion of total factor productivity cannot be partially explained by other factors. For instance, think of efficiency wages, as in Stiglitz 1984. This is not the context for a comprehensive treatment of the topic, but the reader will benefit from the omitted effects framework even when studying production and aggregate supply, as well as the potential externalities (positive or negative) arising from the dynamics of other key variables in the economy.

purpose by applying the notions presented in the first chapter. The second objective is to demonstrate how this framework can effectively address even complex puzzles within a RANK context, providing indications on possible transmission channels not foreseen in the original model.

The puzzles I will attempt to address are twofold: the Forward Guidance Puzzle and the one related to the government spending multipliers. The first pertains to the New Keynesian models' excessive prediction about the real impact of a monetary authority's announcement to adjust policy interest rates further into the future. The baseline representative agent model forecasts a significantly large effect on income, consumption, and inflation from a future reduction in the policy rate. This effect becomes even larger the further into the future the rate cut is expected to take place. However, this is not what has been observed following the implementation of Forward Guidance policies, and the empirical evidence also contradicts this prediction. This issue has been termed the Forward Guidance Puzzle by Del Negro, Giannoni, Patterson, et al. 2012. The aim is to assess whether the introduction of Omitted Effects can cure the puzzle and, if so, to identify which key variable(s) drive the emergence of crucial Omitted Effects.

The second issue has older origins and pertains to the impact of an exogenous increase in government spending on private consumption. The neoclassical school predicts a negative effect of consumption, whereas a segment of the empirical New Keynesian school anticipates a positive impact. This divergence becomes critically important when designing fiscal stabilization policies, particularly in leveraging government spending as a tool to lift the economy out of a recession.

Finally, we will examine how resolving both puzzles simultaneously proves to be a particularly challenging task for most existing models and how this difficulty can be more effectively addressed through the extensive flexibility provided by my framework. Where the RANK model and many of its successors fall short, the model with Omitted Effects succeeds.

As demonstrated in Chapter 3, there are various ways in which Omitted Effects can be introduced into a baseline model: through the households' utility function or the production function. Furthermore, there are multiple approaches to including non-choice variables within the utility function. In the case presented here, I will modify the utility function by making the non-choice variables non-separable only with the consumption component. This approach allows for the consideration of Omitted Effects both within the Euler equation for consumption and in the function defining reservation wages, implying also supply-side effects.

The Omitted Effects I will study are those associated with two key variables: income and inflation. The reasons for considering these two variables are diverse. The assumption underlying my choice is that these variables are the two primary determinants of macroeconomic equilibrium, as represented by the AD-AS model. The pair (y, π) symbolizes the state of the economy, and the joint behaviour of these variables provides insight into whether the economy is in an expansionary or recessionary phase. Additionally, it can reveal the origin of the current business cycle fluctuation, whether it stems from demand-side or supply-side factors. Thus, I implicitly assume that households' utility from consumption endogenously depends on the state of the economy, as represented by the pair (y, π).

The baseline model I modify is a standard vanilla New Keynesian model featuring a representative agent, sticky prices, and monopolistic competition augmented by the inclusion of a government spending shock. We will consider a monetary policy shock and an inflation (cost-push) shock in addition to the fiscal one.

After introducing the RANK model with Omitted Effects, I will analyze its determinacy properties, characterize the conditions that resolve the two puzzles, examine the impulse response functions, and analytically derive the Omitted Effects that characterize the model.

4.1 The Model Economy

Households The economy is inhabited by households that maximize their utility by choosing the optimal quantity of consumption to demand and labour to supply in every period. The objective function takes the following form:

$$U_t = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} Y_t^{\gamma} \Pi_t^{\delta} - \frac{N_t^{1+\eta}}{1+\eta} \right\} \tag{21}$$

Compared to RANK, there is an additional non-separable component alongside the CRRA component of consumption, linking utility to current Output and gross inflation. It implies that the utility of agents, and consequently the marginal utility of consumption, depends on these two gross rates. The latter moderates the contribution of consumption to utility, as captured by the marginal utility of consumption: $\frac{dU_t}{dC_t} = C_t^{-\sigma} Y_t^{\gamma} \Pi_t^{\delta}$. This indicates that the increase in utility due to consumption growth depends not only on the consumption itself but also on the state of the economy, represented by the levels of Output and gross inflation. The novelty in the transmission channels of the economy's dynamics will therefore be a function of the two key parameters, γ and δ, namely the elasticities of the marginal utility of consumption to the two variables.

The intertemporal optimization process consists of maximizing Equation 21 subject to the following intertemporal budget constraint.

$$P_t C_t + B_t = W_t N_t + B_{t-1}(1 + i_{t-1}) + D_t + T_t \tag{22}$$

Where P_t represents the price level, $W_t N_t$ is the wage bill, B_t is the amount of state-contingent bonds, and D_t and T_t are respectively the amount of dividends and transfers received.

The constrained optimization process leads to four first-order conditions including the budget constraint, namely, the condition for consumption, for labour supply, and the demand for state-contingent bonds: $\lambda_t / P_t = C_t^{-\sigma} \Pi_t^{\gamma} R_t^{\delta}$, where λ_t is the Lagrange multiplier; $W_t / P_t = \lambda_t^{-1} N_t^{\eta}$, that links the reservation real wage with the labour supply; $\frac{\lambda_T}{P_t} = \beta \frac{\lambda_{t+1}}{P_{t+1}}(1 + i_t)$ that captures the intertemporal demand for state-contingent bonds and finally the budget constraint itself. Substituting the consumption F.O.C into the one for the bonds and using the Lagrange multiplier inside the labour F.O.C, we get the following two.

$$\frac{C_t^{-\sigma} Y_t^{\gamma}}{P_t} \Pi_t^{\delta} = \beta \frac{C_{t+1}^{-\sigma} Y_{t+1}^{\gamma} \Pi_{t+1}^{\delta}}{P_{t+1}} (1 + i_t) \tag{23}$$

$$\frac{W_t}{P_t} = C_t^{\sigma} Y_t^{-\gamma} \Pi_t^{-\delta} N_t^{\eta} \tag{24}$$

Equations (23) and (24) represent the Euler equation for consumption and the reservation wage. These encapsulate the novelty that will allow us to deviate from the benchmark in terms of normative and positive features[10].

Firms.

We assume that there are two kinds of firms, the final goods producers and the intermediate goods producers. Final goods producers are perfectly competitive, the intermediate goods market

[10]The Euler equation for consumption will consider two additional effects of the two rates, the Omitted Effects, in addition to those already considered in RANK. These two factors also influence the wage equation, implying novelties even at the supply level. The two unforeseen effects are easily detectable by difference compared to the respective equations of the RANK model, $\frac{C_t^{-\sigma}}{P_t} = \beta \frac{C_{t+1}^{-\sigma}}{P_{t+1}}(1 + i_t)$; $\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\eta}$.

is characterized by monopolistic competition. Final goods are produced according to the following production technology: $Y_t = \left(\int_0^1 X_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} dj\right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$. X_t is the amount of intermediate goods used as production input. Costs minimization by final goods firms, taking the price P_t as given brings to the following demand schedule: $X_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} Y_t$, $P_t = \left(\int_0^1 P_t(j)^{1-\epsilon_p} dj\right)^{\frac{1}{1-\epsilon_p}}$. Intermediate good firms produce through a decreasing return production function by using labour as the only input. $Y_t(j) = N_t(j)^{\alpha}$. The marginal productivity of labour can be defined as $MPL_t = \alpha N_t^{\alpha - 1}$. Hence the marginal cost can be defined as $MC_t = \frac{W_t/P_t}{MPL_t}$. Intermediate firms are supposed to not be able to adjust the selling price in every period, but instead, they face a constant probability of setting the price. According to Calvo 1983, this probability is $1-\theta$. That means that in every period a constant fraction of firms is allowed to set the price, while the remaining fraction θ keeps the price level unchanged. The pricing problem can be solved considering the profit maximization procedure of a firm that is allowed to reset the price at time t but doesn't know how many periods it must wait before resetting the price again in the future: $\max_{p_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{Q_{t,t+k}\left(p_t^* Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t})\right)\right\}$. Subject to the sequence of demand constraints $X_{t+k}(j) = \left(\frac{P_t(j)^*}{P_{t+k}}\right)^{-\epsilon_p} Y_{t+k}$.

This brings us to the following First Order Condition:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{\Lambda_{t,t+k} Y_{t+k}(j) \left(\frac{P_t^*}{P_{t+k}} - \mu^p MC_{t+k}\right)\right\} = 0 \tag{25}$$

that in turn leads to the aggregate price level: $P_t = \left[\theta P_{t-1}^{1-\epsilon_p} + (1-\theta)(P_t^*)^{1-\epsilon_p}\right]^{\frac{1}{1-\epsilon_p}}$

Monetary and Fiscal Policy

The government spending takes the form of an autoregressive process of order 1:

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g. \tag{26}$$

Where $\rho_g \in [0, 1]$.

To close the model it is necessary to include a monetary authority that sets the nominal interest rate. This is possible by the introduction of a Taylor rule that sets the nominal rate in response to a current level of the inflation rate different from the target. In this case, we simply assume that this target equals zero.

The monetary policy rule takes the following form:

$$i_t = \phi_\pi \pi_t + \nu_t^i. \tag{27}$$

Where ϕ_π is the Taylor coefficient and ν_t^i is the exogenous component the follows an autoregressive process, augmented with a zero-mean shock.

Market Clearing

The aggregate resource constraint takes the following form:

$$Y_t = C_t + G_t. \tag{28}$$

4.1.1 Steady State and Linearization

The resolution of the model and its linearization lead to a RANK-isomorphic setting. We assume that the steady state is characterized by zero inflation so that Π equals 1. From this, it follows that the steady-state level of the interest rate is equal to ρ. By linearizing equations (23) and (24) around the steady state, we obtain the respective linear equations that determine the dynamics for consumption and reservation wages:

$$c_t = E_t(c_{t+1}) - \frac{1}{\sigma}(i_t - E_t(\pi_{t+1})) - \frac{\gamma}{\sigma}E_t(\Delta y_{t+1}) - \frac{\delta}{\sigma}E_t(\Delta\pi_{t+1}), \tag{29}$$

$$w_t = \sigma c_t + \eta n_t - \gamma y_t - \delta\pi_t. \tag{30}$$

Equations (29) and (30) are the keys to the model determining all the innovative aspects of our model economy. The Euler equation differs from the baseline due to the presence of two terms, $E_t(\Delta y_{t+1})$ and $E_t(\Delta\pi_{t+1})$, equal to $E_t(y_{t+1} - y_t)$ and $E_t(\pi_{t+1} - \pi_t)$ respectively. These two are weighted by γ and δ, the two elasticities of utility to output and the inflation respectively. These elements capture the demand Omitted Effect of the non-choice variables. In the case where γ and δ are both equal to zero, we revert to the baseline RANK, where the Euler equation returns to its baseline version. The two elasticities, $\frac{\gamma}{\sigma}$ and $\frac{\delta}{\sigma}$, represent the effect on consumption of an acceleration of income and inflation. Similar reasoning applies to the wage equation.

Concerning the supply side, the key equations of the model, in linearized form become:

$$y_t = \alpha n_t \tag{31}$$

$$mc_t = w_t + (1-\alpha)n_t \tag{32}$$

$$\pi_t = \beta^\pi E_t(\pi_{t+1}) + \kappa mc_t + \nu_t^\pi \tag{33}$$

Where (31) represents the production technology,(32) the deviations of marginal costs from steady-state costs, and (33) represents the Intertemporal Phillips Curve. κ is a collection of exogenous parameters known in the new Keynesian literature, which account for the Calvo pricing mechanism and the degree of monopolistic competition among firms. Lastly, ε_t^π is an exogenous shock (cost-push shock), with zero mean and constant variance, $\varepsilon_t^\pi \sim N(0, \sigma_\pi^2)$. It follows an autoregressive process of order 1, $\nu_t^\pi = \rho_\pi \nu_{t-1}^\pi + \varepsilon_t^\pi$, with $0 < \rho_\pi < 1$. As in Florin O Bilbiie 2019, we will assume that β^π is equal to 0, expressing the aggregate supply equation solely as a function of marginal costs, enabling us to treat the model as a single equation model. The Phillips Curve will thus take the following form:

$$\pi_t = \kappa mc_t + \nu_t^\pi \tag{34}$$

Finally, the model is closed by the equilibrium condition between supply and demand:

$$y_t = c_t + g_t. \tag{35}$$

Baseline Parameter Calibration

The calibration aligns with the New Keynesian literature. Specifically, a value of 1 is attributed to σ, indicating logarithmic preferences for household consumption. The quarterly intertemporal discount rate is set at 1%, implying a discount factor of 0.99. The Frisch labour elasticity is calibrated to 0.5, consistent with much of the literature. For the autoregressive components of the shocks, I set $\rho = 0.8$.

Since we have a single forward-looking variable, the model can be solved in terms of a single state variable, as all other variables will be functions of it and the exogenous shocks. Therefore,

Table 1: Baseline Parameter Calibration

Parameter	Label	Calibration
β	Discount factor	0.99
γ	Utility Elasticity to Income	$(-\infty, +\infty)$
δ	Utility Elasticity to Inflation	$(-\infty, +\infty)$
ω	Calvo coefficient	0.66
σ	Relative Risk Aversion Coefficient	1
η	Inverse Frisch Labor Supply Elasticity	0.5
ϕ_π	Taylor Coefficient: Inflation	1.5
ϕ_y	Taylor Coefficient: Output	0.5
ρ_g	Autoregressive Coefficient: Government Spending	0.1, 0.5, 0.9
ρ_i	Autoregressive Coefficient: Monetary Policy	0.1, 0.5, 0.9
ρ_π	Autoregressive Coefficient: Cost Push	0.1, 0.5, 0.9

Table 1: Baseline Parameter Calibration

I substitute the wage equation, the Phillips curve, the Taylor rule, and the three shocks into Equation (2), obtaining the following state-space representation.

$$c_t = \Omega_c E_t c_{t+1} + \Omega_g g_t - \Omega_\pi \nu_t^\pi - \Omega_i \nu_t^i \tag{36}$$

Where:

$$\Omega_c = \frac{1-\gamma+(1-\delta)\chi_\pi(\sigma+\eta-\gamma)}{1-\gamma+(\phi_\pi-\delta)\chi_\pi(\sigma+\eta-\gamma)};\Omega_g = \frac{\gamma(1-\rho_g)-(\phi_\pi-1)\chi_\pi(\eta-\gamma)}{1-\gamma+(\phi_\pi-\delta)\chi_\pi(\sigma+\eta-\gamma)}$$

$$\Omega_\pi = \frac{(\phi_\pi-1)\chi_\pi}{1-\gamma+(\phi_\pi-\delta)\chi_\pi(\sigma+\eta-\gamma)};\Omega_i = \frac{1}{1-\gamma+(\phi_\pi-\delta)\chi_\pi(\sigma+\eta-\gamma)}$$

are convolutions of exogenous parameters.

4.2 Equilibrium Determinacy, Sargent-Wallace and the Forward Guidance Puzzle

In this section, I will analyze the determinacy properties of the augmented model, some of which will enable us to address the Forward Guidance Puzzle.

According to O. J. Blanchard and Kahn 1980, the eigenvalues of the transition matrix must lie outside the unit circle for the model to be stable and return to its steady state following a perturbation. However, in our case, we focus solely on Equation (36). Therefore, rather than dealing with a matrix, we consider a single coefficient, Ω_c, which must satisfy $-1 < \Omega_c < 1$.

In the RANK framework, the condition ensuring a unique local equilibrium is $\phi > 1$, meaning that the monetary policy rule must meet the Taylor principle. However, by incorporating the Omitted Effects of income and inflation, the issue becomes complex and less intuitive. In this last case, since no restrictions are imposed on the parameters γ and δ, which are the key parameters, the rule that ensures equilibrium determinacy requires the simultaneous satisfaction of the following two conditions:

$$\frac{(1-\phi_\pi)(\sigma+\eta-\gamma)}{1+\chi_\pi\delta} < 0 \ and \ \phi_\pi > \phi_\pi^* \longleftrightarrow \phi_\pi^* \equiv \frac{2(\gamma-1)}{\sigma+\eta-\gamma}(1+\chi_\pi\delta)+2\delta-1. \tag{37}$$

We can identify four distinct circumstances that represent the complexity: first, in the case where $\gamma > \sigma + \eta$ and $\delta < -1/\chi$, and in the opposite case, where $\gamma < \sigma + \eta$ and $\delta < -1/\chi$, the condition that ensures determinacy equilibrium is as follows:

$$\phi_\pi^* < \phi_\pi < 1. \tag{38}$$

In the two complementary cases, however, equilibrium requires the fulfilment of the following:

$$\phi_\pi > argmax(1, \phi_\pi^*). \tag{39}$$

Determinacy equilibrium depends on the relative strength of the two channels, one linked to income and the other to inflation. First, note that the convolution $\gamma \lesseqgtr \sigma + \eta$, which is crucial in determining one case or the other, is related to deep parameters concerning agents' utility. Therefore, we can say that the income-related channel, identified by γ, is associated with demand-side factors. In contrast, the parameter δ, connected to the Omitted Effect of inflation, is tied to κ, a supply-side term. Indeed, κ converts marginal costs into inflation through sticky prices and monopolistic competition.

The Taylor principle may not be sufficient in one case, or it may even lead to disequilibrium in another. Indeed, as shown in condition 37, under certain circumstances, the combination of Omitted Effects necessitates a policy interest rate adjustment that is less than a one-to-one to inflation. However, if $\phi_\pi^* < 1$, the Taylor principle becomes sufficient.

A particular circumstance in which it is useful to study equilibrium requirements is the Sargent-Wallace case. In Sargent and Wallace 1976, the authors analyze economic equilibrium in the scenario where monetary policy is decoupled from the business cycle, and specifically from inflation dynamics. The case in which monetary policy does not react to the business cycle can be represented by the interest rate peg, where $\phi_\pi = 0$. The Sargent-Wallace conditions become crucial when studying the behaviour of the economy near the Effective Lower Bound of the policy interest rate, or when examining the effectiveness of unconventional monetary policies such as the Forward Guidance.

After hitting the zero lower bound on the policy interest rate, the U.S. Federal Reserve, the European Central Bank, and the Bank of Japan have experimented with interest rate announcement policies. Unable to lower rates further, forward guidance involves signaling that interest rates will remain low or decrease in the future to influence the real economy today. However, the RANK model, along with other benchmark models—such as those of L. J. Christiano, Eichenbaum, and Evans 2005 and Smets and Wouters 2007—predicts a response of consumption and inflation to such announcements that deviates anomalously from observed patterns. To show under which conditions the forward guidance puzzle can be resolved, I begin with the general condition that examines the impact of a shock on the nominal interest rate, which does not occur in the current period, but takes place in a future period T.

$$c_t = \Omega_c^T E_t c_{t+T} - \Omega_i E_t \sum_{j=0}^{T-1} \Omega_c^j \nu_{i,t+j} \tag{40}$$

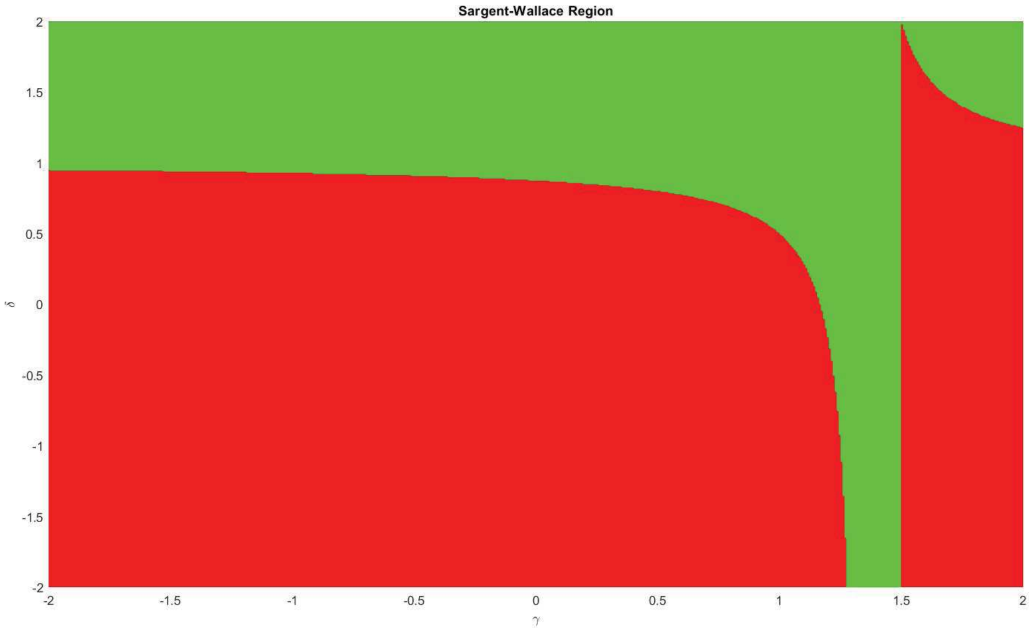


Figure 1: Determinacy Equilibrium conditions under interest rate peg in terms of gamma and delta. X-axis: gamma, Y-axis: delta. Green area: determinacy region. Red area: indeterminacy region. Parameters calibration: $\sigma = 1$; $\eta = 0.5$; $\chi_\pi = 0.5$; $\phi_\pi = 0$.

In general, the value of Ω_c determines the asymptotic behaviour of consumption following a shock that occurs at time T. Specifically, if the absolute value of Ω_c is between -1 and 1, the effect of a shock that occurs at time T tends to zero as T approaches infinity. To close the forward guidance puzzle, it is necessary for Ω_c to lie between -1 and 1, with the additional condition that ϕ_π must equal zero, meaning there should be no monetary adjustment. This is exactly what happens during forward guidance: an announcement that shapes future interest rate expectations without any monetary intervention until time T.

The Forward Guidance Puzzle is solved if and only if:

$$-1 < \frac{1 - \gamma + \frac{1-\delta}{1+\chi_\pi \delta}(\sigma + \eta - \gamma)}{1 - \gamma - \frac{\delta}{1+\chi_\pi \delta}(\sigma + \eta - \gamma)} < 1. \tag{41}$$

In Figure 1, the combinations of γ and δ that ensure equilibrium determinacy under the interest rate peg are shown. It should be noted that in the baseline, no combination of permissible parameters guarantees such a condition. This is precisely what gives rise to the macroeconomic puzzle. This means that the mere existence of a solution to condition 41 ensures that the RANK model with omitted income and inflation effects represents a solution to the puzzle. Furthermore, as shown in the figure, for positive values of δ, approximately greater than 1, equilibrium determinacy is ensured. This holds for a wide range of γ values. This seems to suggest that although both Omitted Effects are relevant to cure the puzzle, the one of inflation plays a prominent role. Furthermore, the solution requires largely positive values of δ, implying an expansive role for unexpected inflation on consumption and a recessive role for expected inflation. Such a picture assigns inflation the role of a key variable in solving the forward guidance puzzle.

In the literature, there are many approaches to tackling the Forward Guidance (FG) Puzzle.

McKay, Nakamura, and Steinsson 2017, as well as Hagedorn et al. 2019, refer to incomplete markets to obtain a state-space representation capable of resolving the dilemma. Nakata et al. 2019 emphasize the importance of a discounted Euler equation to fix the puzzle. The same approach is found in Florin Ovidiu Bilbiie 2018, where agents heterogeneity and the presence of idiosyncratic risk enable Euler discounting. This framework is also central to the contribution of McKay, Nakamura, and Steinsson 2016, who apply money in the utility function to derive an Euler curve in consumption that discounts future expectations. Solutions to the puzzle are also provided through the cognitive discounting of Gabaix 2020, heterogeneous beliefs of Andrade et al. 2019, Perpetual Youth of Del Negro, Giannoni, Patterson, et al. 2012, and imperfect communication of J. R. Campbell et al. 2019.

The difference in the approach of the Omitted Effects lies primarily in the fact that the underlying framework is always the same: RANK. This represents an advantage as if we are able to solve a puzzle without changing the framework, it may indicate a weakness in the RANK model, which, once resolved, will likely allow us to better understand the economy and predict its future behaviour. Moreover, by identifying the omitted inflation effect as the key transmission mechanism in solving the puzzle, we also gain an indication of where economic research should focus, namely, in better understanding the role of inflation in influencing consumption and the broader economy.

4.3 Impulse Responses, Omitted Effects and the Government Spending Puzzle

In this section, I address three key topics: impulse responses, Omitted Effects, and the conditions under which the government spending puzzle is resolved. Furthermore, we go beyond this analysis to examine the conditions that resolve the same under the interest rate peg.

Starting from Equation (36) and applying the method of undetermined coefficients, we derive a general solution for the impulse responses. Given the presence of only one state variable, the impulse response function for each shock can be represented by a single equation. Solving for a generic shock j, the response of aggregate consumption becomes:

$$c_t = \left(\frac{\Omega_{\varepsilon,j}}{1 - \Omega_c \rho_j} \right) \varepsilon_{t,j} \tag{42}$$

For convenience, and given that the purpose is both demonstrative and aimed at resolving the fiscal puzzle, we will focus on the impulse response functions.

Solving equation (36) for the three shocks—government spending, inflation, and the interest rate—we obtain:

$$c_t = \frac{\gamma(1-\rho_g) - (\phi_\pi - 1)\frac{1}{1+\chi\delta}(\eta - \gamma)}{(1-\rho_g)(1-\gamma+(1-\delta)(\frac{1}{1+\chi\delta}(\sigma+\eta-\gamma))} g_t \tag{43}$$

$$c_t = -\frac{\frac{(\phi_\pi - 1)}{1+\chi\delta}}{(1-\rho_\pi)(1-\gamma+(1-\delta)(\frac{1}{1+\chi\delta}(\sigma+\eta-\gamma))} \nu_{t,\pi} \tag{44}$$

$$c_t = -\frac{1}{(1-\rho_i)(1-\gamma+(1-\delta)(\frac{1}{1+\chi\delta}(\sigma+\eta-\gamma))} \nu_{t,i} \tag{45}$$

In addition to the deep parameters and the autoregressive coefficients, the consumption responses to the shocks are functions of the monetary policy rule, captured by the Taylor coefficient

ϕ_π, and of the two Omitted Effects, captured by the parameters γ and δ. We are now ready to define analytically the Omitted Effects, as introduced in Chapter 1, related to the consumption responses.

First, there is the overall Omitted Effect, which manifests as the difference in the key coefficients between the modified model and the baseline model that assumes the parameters γ and δ are equal to zero. Additionally, there is an Omitted Effect for each non-choice variabl. In our case, this includes the effect of inflation, captured by δ, and the one of income, captured by γ. Lastly, there is the effect of the interaction between the two non-choice variables, which we leave in residual form. Focusing on the impulse response to a government spending shock g, we obtain:

$$\mathcal{O} = \frac{\gamma(1-\rho_g) - (\phi_\pi - 1)\frac{1}{1+\chi\delta}(\eta-\gamma)}{(1-\rho_g)(1-\gamma+(1-\delta)(\frac{1}{1+\chi\delta}(\sigma+\eta-\gamma))} + \frac{(\phi_\pi-1)\eta}{(1-\rho_g)(1+\sigma+\eta)} \tag{46}$$

$$\mathcal{O}_{\gamma|\delta=0} = \frac{\gamma(1-\rho_g) - (\phi_\pi - 1)(\eta-\gamma)}{(1-\rho_g)(1-\gamma+(1-\delta)(\frac{1}{1+\chi\delta}(\sigma+\eta-\gamma))} + \frac{(\phi_\pi-1)\eta}{(1-\rho_g)(1+\sigma+\eta)} \tag{47}$$

$$\mathcal{O}_{\delta|\gamma=0} = \frac{(\phi_\pi - 1)\frac{1}{1+\chi\delta}\eta}{(1-\rho_g)(1+\sigma+\eta)} - \frac{(\phi_\pi-1)(\eta-\gamma)}{(1-\rho_g)(1-\gamma+(1-\delta)(\frac{1}{1+\chi\delta}(\sigma+\eta-\gamma))} \tag{48}$$

$$\mathcal{O}_{\gamma\delta} = \mathcal{O} - \mathcal{O}_{\gamma|\delta=0} - \mathcal{O}_{\delta|\gamma=0} \tag{49}$$

Equations (46 - 49) represent the analytical definitions of Omitted Effects. The term on the right-hand side of the first three equations represents the response of consumption to government spending in the baseline RANK framework.

We now turn to the condition that solves the government spending puzzle. The solution requires the following:

$$\gamma > argmax\left(\frac{(\phi_\pi - 1)\frac{1}{1+\chi\delta}\eta}{(1-\rho_g+(\phi_\pi-1)\frac{1}{1+\chi\delta}}, \frac{1+(1-\delta)\frac{1}{1+\chi\delta}(\sigma+\eta)}{1+(1-\delta)\frac{1}{1+\chi\delta}}\right) \tag{50}$$

Alternatively, γ must be smaller than the minimum argument.

To simplify the discussion, Figures 2 and 3 illustrate the combinations of γ and δ, given different values of ϕ_π, that satisfy the dynamic equilibrium condition while simultaneously ensuring a positive response of aggregate consumption following a government spending shock. The reported results correspond to four different values of the Taylor coefficient, ϕ_π. The bottom-right case represents a particularly noteworthy scenario that requires special attention: the interest rate peg.

Two distinct figures have been provided to represent the same concept: in Figure 2, we focus on a narrower domain of the two coefficients by zooming in for greater detail.

The first crucial point to emphasize is that by introducing these Omitted Effects, there exists at least one admissible combination of parameters that resolves the fiscal puzzle. This stands in contrast to the standard RANK model, where no solution exists. In the model where the Omitted Effects of inflation and income play an active role, it becomes possible to achieve a positive consumption response to government spending.

By paying attention to the various combinations, the reader will notice that, as in the case of equilibrium determinacy discussed earlier, the coefficient that seems to drive the result is δ, which represents the elasticity of the utility function and also the marginal utility of consumption with respect to the gross inflation rate. Indeed, beyond the complexity of admissible regions, it appears that δ is the decisive parameter in determining the response of consumption to government

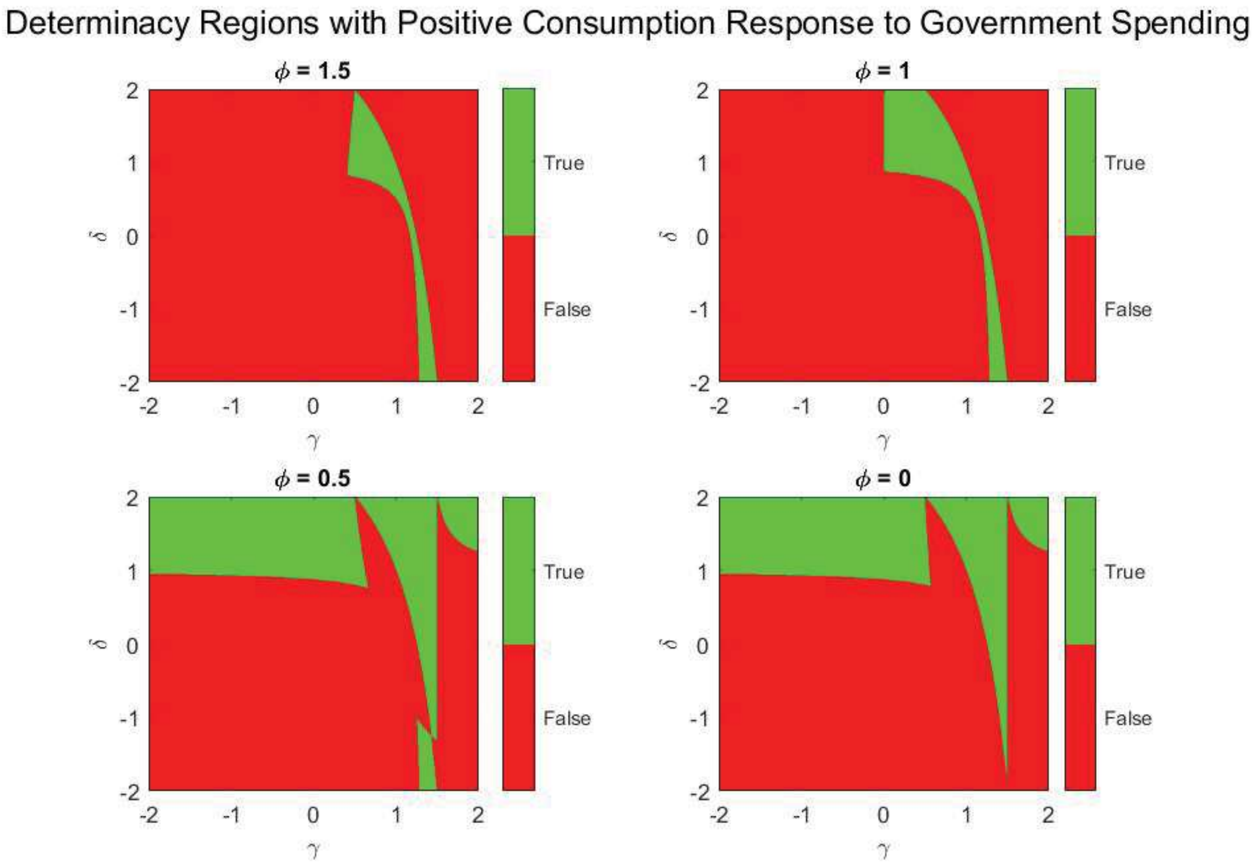


Figure 2: Determinacy regions of γ and δ for four cases based on the values of ϕ (1.5, 1, 0.5, and 0, corresponding to the Sargent-Wallace case). The other parameters are calibrated as follows: $\sigma = 1$, $\eta = 0.5$, $\chi = 0.5$, and $\rho_g = 0.8$. The green area represents the regions where the consumption response to government spending is positive and compatible with determinacy equilibrium, while the red area indicates regions where one of the two conditions is not satisfied. The domain is $[-2, 2]$ for both γ and δ.

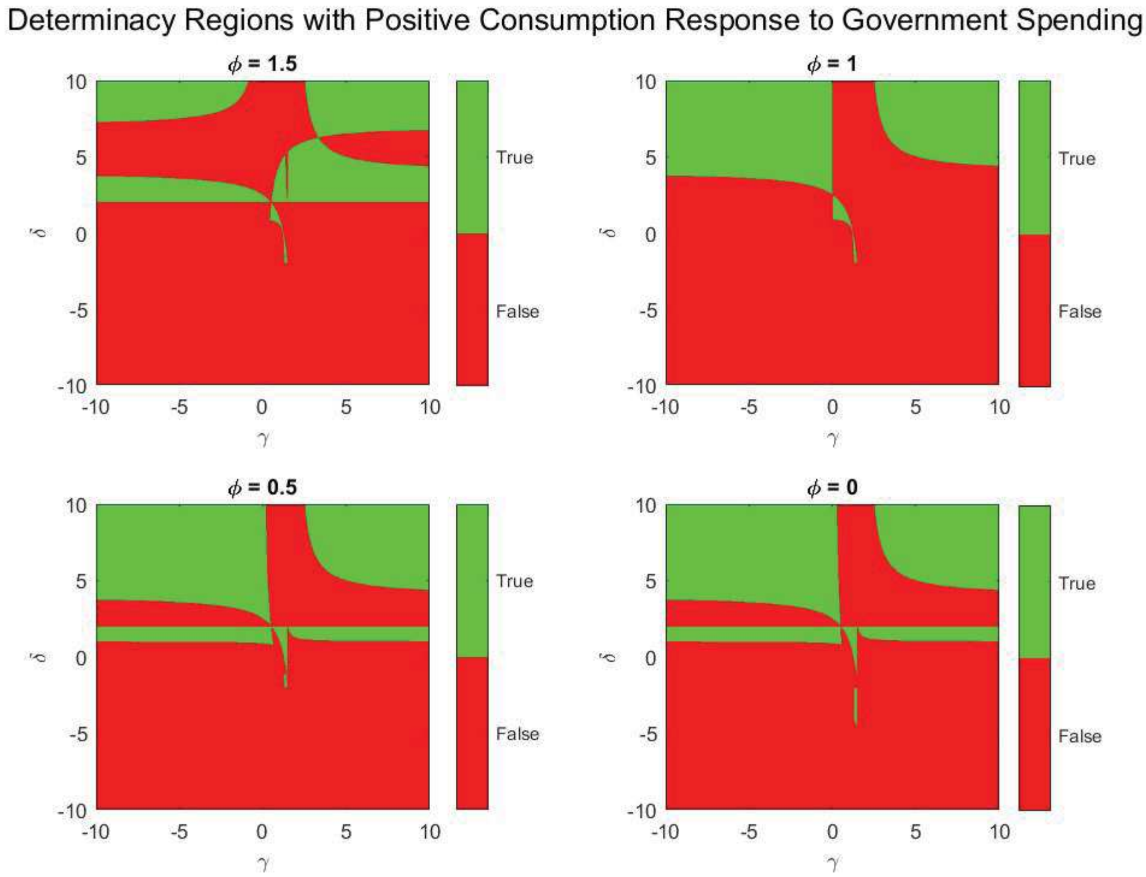


Figure 3: Determinacy regions of γ and δ for four cases based on the values of ϕ (1.5, 1, 0.5, and 0, corresponding to the Sargent-Wallace case). The other parameters are calibrated as follows: $\sigma = 1$, $\eta = 0.5$, $\chi = 0.5$, and $\rho_g = 0.8$. The green area represents the regions where the consumption response to government spending is positive and compatible with determinacy equilibrium, while the red area indicates regions where one of the two conditions is not satisfied. The domain is $[-10, 10]$ for both γ and δ.

spending, given, of course, the behaviour of the fiscal rule. Specifically, when focusing on the last case, the interest rate peg, where $\phi_\pi = 0$, it is evident that for $\gamma = 0$ and for nearly all values of $\delta > 1$, the puzzle is cured. We can go further. What has been said not only allows us to resolve the government spending puzzle but also the Catch-22, which refers to the solution of the fiscal puzzle under the interest rate peg. In our model, it seems that explaining the Catch-22 is easier than achieving the same result in the presence of an aggressive rather than accommodative monetary policy.

At this point, if inflation is the key variable, it is important to understand the transmission mechanisms that allows for this positive consumption response. It should be noted that we cannot disregard the way in which the Omitted Effects were introduced. In our example, the objective function was modified in such a way that the additional effects were not separable only concerning household consumption. Through the optimization process, this altered both the intertemporal IS curve and the wage-setting schedule. It is precisely from the interaction between these two effects that the result did emerge. From a consumption perspective, when δ is positive and sufficiently large, a direct shock or one that indirectly impacts inflation, has the consequence of increasing consumption. In a model where complete microfoundation is assumed, this happens, for example, when money or liquidity is introduced into the utility function. However, the expansionary effect of contemporaneous inflation is not the only key element. In fact, what happens in the labour market is even more determinant. If δ is positive and sufficiently large, this means that an increase in inflation at time t has a depressive effect on real wages. The reduction in real wages in the RANK model reduces marginal costs, generating a decrease in the nominal interest rate. Moreover, as wages decrease, labour demand increases and production rises. In other words, the inflation-related channel is the determining factor in generating a positive consumption effect. None of this could happen if the Omitted Effects of inflation and income, in addition to triggering a response in consumption and wages as explained, also allowed for a substantial relaxation of the conditions for determinacy, preventing an explosive dynamics of macroeconomic variables far from equilibrium.

5 Estimation of the Omitted Effects

In this section, the estimation of Omitted Effects will be carried out in an empirical context. Transitioning from theory to practice is essential for several reasons. First, it is useful to apply the model to test the validity and significance of the framework. Furthermore, I propose the estimation of Omitted Effects as a preferred and robust tool for performing the selection of the omitted variables to be considered. Every process leading to the formulation of a model's hypotheses is prone to errors, inaccuracies, and biases, the effects of which can go beyond those initially expected by the economist. This can lead to puzzles that do not reflect empirical evidence and more.

In the following, I will perform a recursive estimation, aiming to identify the Omitted Effects that improve the model's fit to the data. The algorithm is reported in Appendix A.2 and involves introducing endogenous variables and their corresponding estimated coefficients into some of the key equations of the model, based on a certain microfoundation.

5.1 Empirical Results

I will focus on the U.S. economy and use macroeconomic data from March 1969 to December 2019. The data source is the FRED database from the Federal Reserve Bank of St. Louis. The variables under examination include real GDP collected at quarterly frequencies, aggregate consumption of non-durable goods by American households, the Consumer Price Index (CPI), the federal funds interest rate on a quarterly basis, and federal government spending. To perform a business cycle analysis, all variables, except for the nominal policy interest rate, were filtered using a Hodrick-Prescott filter. In Figure 4, the detrended and seasonally adjusted time series are reported, except for the nominal interest rate, which is used as-is. Table 2 provides a summary of the key characteristics of the series.

Table 2: Descriptive Statistics

Statistic	y	c	i	π	g
Mean	0	0	0	0.009	0
Std Dev	0.014	0.011	0.067	0.006	0.011
10th Percentile	-0.017	-0.014	-0.077	0.003	-0.014
Median	0	0.000	0.010	0.007	0
90th Percentile	0.016	0.014	0.080	0.018	0.014

Notes: Descriptive Statistics for Variables y, c, i, π, and g

The estimation involves the benchmark model, RANK.

The presence of a rational representative agent, monopolistic competition, and sticky prices make it an ideal neutral environment for testing our hypothesis regarding Omitted Effects. As noted in Chapter 3, there is no single way to introduce Omitted Effects, as they can be incorporated into any objective function, such as the household utility function, the firm production function, or even monetary and fiscal policy rules. Furthermore, even within a single objective function, there are multiple approaches to address omitted variables, considering the variety of possible functional forms and combinations. This flexibility depends on the judgment of the economist conducting the test.

I will modify only two equations—the aggregate consumption equation and the labour supply equation. This approach is based on the premise that introducing Omitted Effects exclusively into the household utility function directly impacts only consumption and wages. This does not preclude the propagation of these effects throughout the general equilibrium.

The invariant component of the model comprises equations in table 3. In addition to those, I will introduce the two equations derived from the utility optimization process incorporating Omitted Effects. The variables constituting the model are Aggregate Consumption c_t, Output y_t, Government Consumption g_t, Policy Interest rate i_t, Inflation π_t, Employment n_t, the Real Wage w_t.

Concerning parameter calibration, refer to the table **??** in Chapter 4, except the autoregressive coefficient of the shocks, which is set here to 0.8.

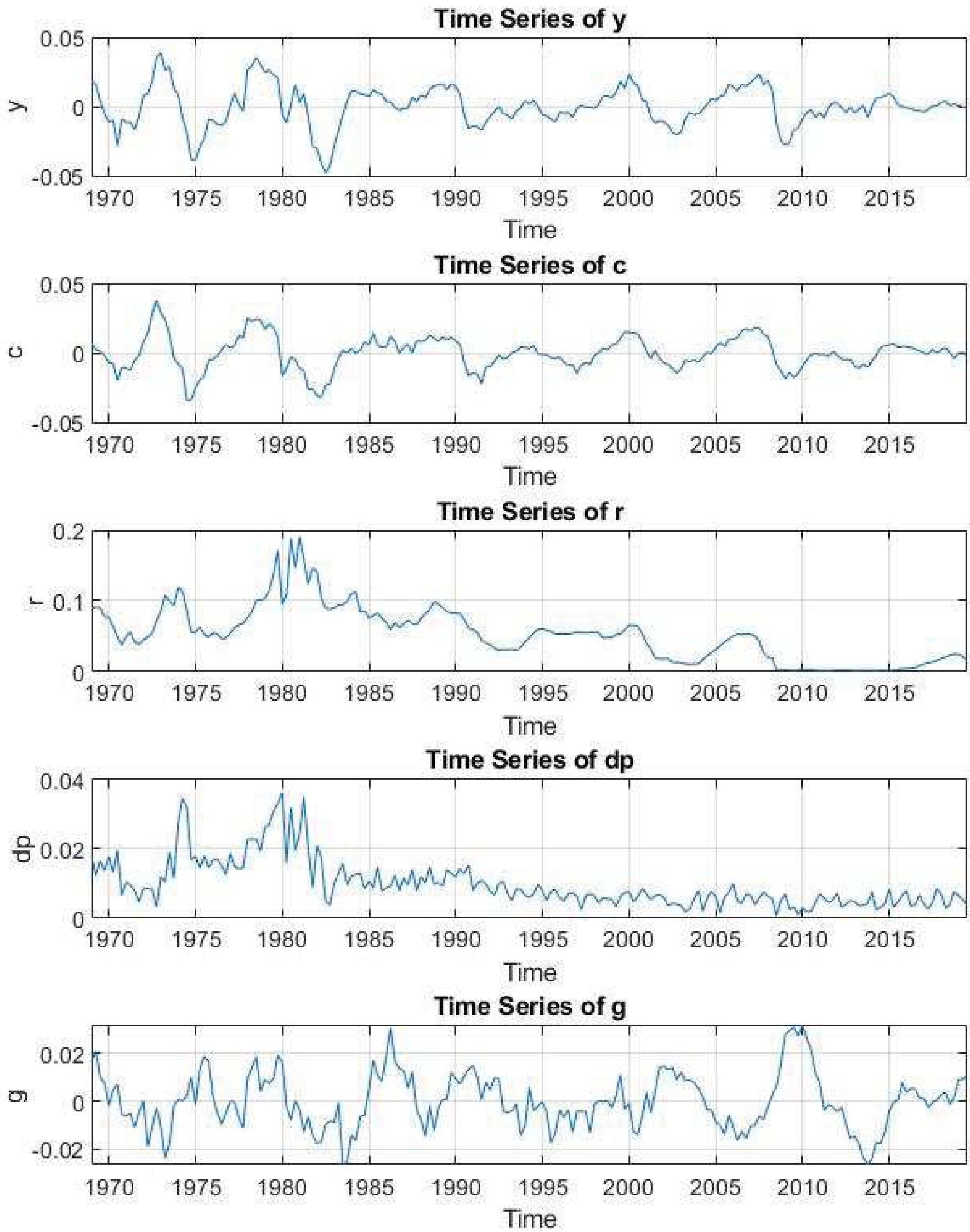
5.2 Discussion

The Omitted Effects are incorporated into the model through the use of an Augmented Objective Function, which takes the following form.

$$U_t^{OE} = U_t \prod_{j=1}^{m} \mathbf{X_{j,t}}^{\gamma^j}$$

In our case, the utility function takes the form shown in the following equation.

Figure 4: Time Series for US Economy



Note: Time Series for US Economy. 1969(Q1) - 2019(Q4). Hodrick-Prescott filtered series of Real GDP (y), Aggregate Consumption (c), Federal Funds Rate (r), CPI Inflation Rate (dp $=\pi$), Federal Government Consumption (g).

Table 3: Equations of the DSGE Model

Equation	**Description**
$\pi_t = \beta E_t(\pi_{t+1}) + \kappa mc_t + \epsilon_t^{\pi}$	Phillips curve with forward-looking expectations
$mc_t = w_t + (1-\alpha)n_t$	Marginal costs
$y_t = a_t + \alpha n_t$	Production function
$i_t = \phi_\pi \pi_t + \phi_y y_t + u_t^i$	Taylor rule for monetary policy
$y_t = c_t + g_t$	Aggregate resource constraint
$g_t = \rho_g g_{t-1} + \epsilon_t^g$	Process for government spending
$a_t = \rho_a a_{t-1} + \epsilon_t^a$	Productivity shock process
$\rho_t = \rho_\rho \rho_{t-1} + \epsilon_t^\rho$	Preference shock process
$u_t = \rho_u u_{t-1} + \epsilon_t^u$	Labor market shock process
$v_t = \rho_v v_{t-1} + \epsilon_t^v$	Other structural shock process

$$U_t^{OE} = \sum_{k=0}^{\infty} \beta^k \left(\frac{C_{t+k}^{1-\sigma}}{1-\sigma} \prod_{j=1}^{m} \mathbf{X_{j,t+k}}^{\gamma^j} - \frac{N_{t+k}^{1+\eta}}{1+\eta} \right) \tag{51}$$

The non-choice variables are incorporated in the form of a Cobb-Douglas combination, multiplying only the consumption component. The budget constraint takes the usual form:

$$P_t C_t + B_t = W_t N_t + B_{t-1}(1 + i_{t-1}) + D_t + T_t \tag{52}$$

The log-linear solution to the household's problem yields the following intertemporal IS curve:

$$c_t = E_t(c_{t+1}) - \frac{1}{\sigma}(i_t - E_t(\pi_{t+1}) - \rho_t) - \sum_{j=0}^{m} \frac{\gamma^j}{\sigma} \left(\mathbf{x}_{j,t+1} - \mathbf{x}_{j,t}\right), \tag{53}$$

and the following wage schedule:

$$w_t = \sigma c_t + \eta n_t - \sum_{j=0}^{m} \gamma^j \mathbf{x}_{j,t}. \tag{54}$$

Equations 53 and 54 complete the model. Following the selection algorithm, all possible combinations are calculated among the 7 variables considered to be potential sources of misspecification. For each partition, the Bayesian estimation of the γ^j coefficients is performed using a non-informative prior distribution (a Uniform prior distribution centred at zero). The log-likelihood is then calculated, and the combinations are ordered descending based on their log-likelihood value.

Table 4 presents the results for the top 10 combinations of Omitted Effects with the highest log-likelihood values. I focus on the first combination and analyze the properties of this model, particularly in the context of the impulse response analysis.

The estimation reveals that the three Omitted Effects which provide the greatest improvement in the model's ability to fit the data pertain to real wages (w), inflation (π), and output (y), with a maximum log-likelihood value of 2455.11. When compared to the log-likelihood of the RANK model without additional effects, which is 2011.89, this represents an improvement of over 24%.

To investigate the impact of these unforeseen effects, it is helpful to begin by rewriting both the consumption equation and the wage equation.

$$c_t = E_t(c_{t+1}) - i_t + E_t(\pi_{t+1}) - \rho_t + 1.49 * (w_{t+1} - w_t) - 0.48 * (\pi_{t+1} - \pi_t) - 0.59 * (y_{t+1} - y_t)$$

$$w_t = c_t + 0.5 * n_t + 1.49 * w_t - 0.48 * \pi_t - 0.59 * y_t$$

Table 4: Estimated Coefficients of Omitted Effects in RANK

Omitted Effects	**Estimated Coefficients**	**Log-Likelihood**
w, π, y	-1.24, 0.48, 0.59	2455.11
c, y, r	-3.20, 0.74, 1.39	2454.98
c, g, r	-2.46, 0.74, 1.39	2454.98
g, y, r	3.20, -2.46, 1.39	2454.98
g, w, r	0.67, -0.78, 0.52	2454.51
c, r	-2.47, 1.37	2453.22
g, π, y, r	3.13, 0.24, -2.40, 1.36	2452.78
c, g, π, r	-2.40, 0.73, 0.24, 1.36	2452.78
g, w, π, a	0.52, -1.36, 0.75, 0.39	2452.78
c, w, π, y	1.96, -1.20, -0.29, 0.65	2452.78

Examining the estimated new IS curve reveals an interesting picture. Concerning the relationship between consumption and wages, it is clear that consumption depends positively on expectations of future wages and negatively on contemporaneous. This suggests the presence of a channel tied to the labour market and wage expectations: agents tend to anticipate and increase consumption if they expect future wage growth. Specifically, what matters is the acceleration in wages, $w_{t+1} - w_t$. In other words, consumption rises if the expected growth in tomorrow's wages overcomes the growth in today's wages.

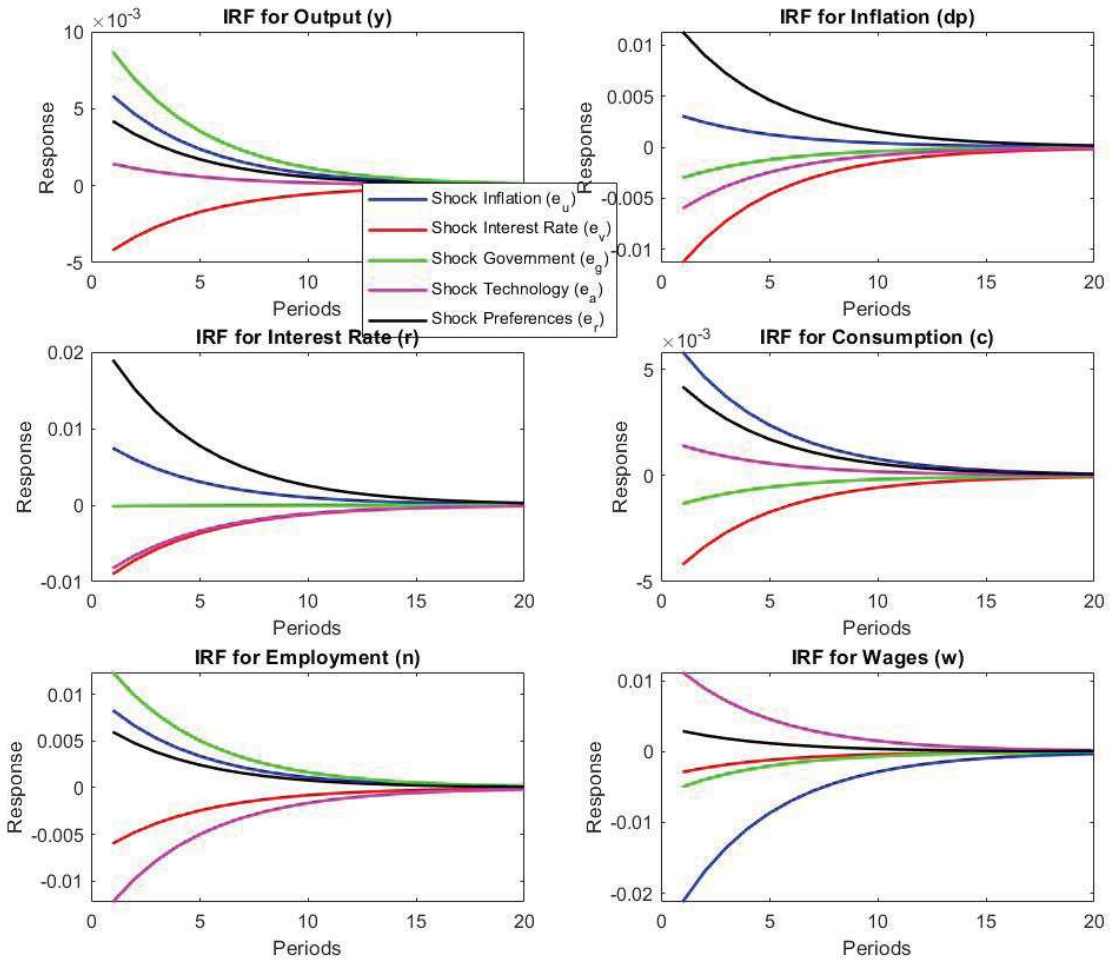
Concerning inflation, consumption is positively related to current inflation but negatively tied to future inflation levels: in this case, consumption decreases if an acceleration in the inflation rate is anticipated. Finally, an expected acceleration in income appears to be associated with a reduction in consumption.

Figure 5 presents the impulse response functions for the variables under study to five shocks: the one of inflation, nominal interest rate, government spending, technology, and preferences.

While all the shocks exhibit dynamics similar, at least in sign, to those predicted by the baseline model, the inflation shock, represented with a blue solid line in Figure 5, positively impacts both consumption and income. This is observed despite a policy interest rate shock producing the usual negative effect on consumption. There seems to be evidence of an upward-sloping demand curve, at least regarding consumption and inflation in a cost-push shock. Thus, this context does not align with the Inverted Aggregate Demand Logic (IADL) of Florin O Bilbiie 2008 and Florin O Bilbiie and Straub 2013 but suggests the presence of another mechanism. Moreover, the cost-push shock, in addition to exerting an expansionary effect on the economy, has a significant negative impact on real wages, but a strong positive effect on employment.

When examining the IRFs related to a monetary shock (red solid line), it becomes evident that the sign of the shock's impact on the variables under consideration aligns with that of the mainstream model: following a nominal policy interest rate shock, the economy contracts, registering a decline in income, consumption, inflation, employment, and wages.

Figure 5: Impulse Responses for Best Log-Likelihood Model



Note: Impulse response functions for inflation, interest rate, public spending, technology, and preference shocks. The IRFs pertain to the variables y, π, r, c, n, and w. The simulated model includes the three omitted effects concerning wages (w), inflation (π), and output (y).

The second notable element is the limited crowding-out of consumption in response to an increase in government spending (green solid line). The RANK model, which predicts a sharper reduction in consumption than observed here, contrasts with the results obtained. While in RANK, a government spending shock has a weakly expansionary effect on output due to the crowding out of consumption, in this case, the reduction in consumption, though still present, is much more limited. As a result, the impact of government spending on income is slightly below unity. Moreover, in our model, employment benefits significantly from the increase in government spending, and wages decrease only modestly. Another interesting aspect is that inflation decreases rather than increases.

In conclusion, consider the IRFs related to a technology shock (pink solid line) and those associated with a preference shock (black solid line). Regarding the technology shock, while it positively affects income and consumption, its impact is significantly weaker than in the RANK model. Moreover, the decline in employment—commonly referred to as the productivity-employment puzzle—is even more pronounced here than in RANK. However, wages rise more in our model compared to the baseline. Similarly, the preference shock follows a pattern consistent with the baseline model but has a stronger effect on inflation and the policy interest rate.

Although we have employed the combination of omitted variables with the highest log-likelihood value as a reference, readers are encouraged to test the application of the other proposed models themselves. All the estimates for the various combinations of Omitted Effects are reported in Appendix B.

Through this estimation exercise, supported by an iterative estimation algorithm, we obtained results that can be summarized as follows. First and foremost, all models incorporating Omitted Effects demonstrated superior performance in fitting the data compared to the benchmark model. This is not surprising, particularly because the other coefficients in the RANK model were calibrated rather than estimated. Thus, it was plausible that any estimated model would outperform the benchmark. However, what is less obvious is the selection of Omitted Effects that perform better than others. In this regard, we have verified that, within the scope of our formulation, the three effects that led to the greatest improvement in log-likelihood are real wages, inflation, and income. This provides a clear indication of the RANK model's weaknesses: alongside the predictable Omitted Effect of income, the inclusion of other features related to inflation and real wages is both intriguing and innovative. This result warrants further investigation to uncover the underlying microeconomic causes, which would enable a comprehensive and coherent reformulation of the model.

As an intermediate research tool, the model with estimated effects proves to be both powerful and useful.

It is, however, important to reiterate four crucial points regarding the work we have undertaken:

1. **Alternative Approaches to Introducing Omitted Effects:** The reader will observe that there is no single way to incorporate Omitted Effects into a DSGE model. In our case, we modified the utility function by superimposing the product of non-choice variables exclusively on consumption. Alternatively, the same could be done for the labour supply, or for both. The combinations are numerous, and we encourage readers to experiment with different configurations, which may lead to further advancements in the analysis.

2. **Applicability Beyond the RANK Model:** While we used the RANK model as a benchmark, this does not imply that the methodology is limited to this specific framework. Indeed, the purpose of this approach is to identify weaknesses and enrich any model, irrespective of its complexity. It would be particularly interesting to include Omitted Effects into more sophisticated frameworks and to investigate which effects might improve their fit to the data and the understanding of economic facts.

3. **Flexibility in Functional Forms and Combinations:** The use of a Cobb-Douglas functional form for introducing unforeseen effects does not preclude the application of alternative functional forms or other types of combinations. Exploring these alternatives could further enrich research and contribute to the scientific discourse.

4. **Alternative Estimation Methods:** While we employed a Bayesian estimation, this does not exclude the use of other methodologies, whether frequentist or otherwise. Additionally, transitioning to the estimation of non-linear models or employing Time-Varying or Markov-Switching features could represent an exciting avenue for advancing the study of Omitted Effects in macroeconomic modelling.

6 Conclusions

In this work, I introduced a novel tool to address research in macroeconomics. This entails the introduction of Omitted Effects within DSGE models. The complexity of dynamic general equilibrium models, coupled with the emergence of puzzles—i.e., discrepancies between the implications of a particular model or class of models and empirical evidence—raises a significant misspecification problem in general equilibrium that is hard to tackle.

By modifying one or more objective functions, I incorporate Omitted Effects into key model equations. This modification is achieved through the introduction of the Augmented Objective Function, which involves the modification of an objective function to include, in a non-separable form with respect to at least one choice variable, other endogenous or exogenous variables.

The work is organized into three main sections. In the first, all the new concepts are introduced, including the Augmented Objective Function and the Omitted Effect.

In the second, I illustrate how starting from a simple baseline vanilla new Keynesian model, the introduction of two Omitted Effects—related to inflation and income—derived from the consumption equation of the representative agent, enables the resolution of both the government spending puzzle and the forward guidance puzzle.

In the final chapter, I follow the empirical approach. Employing an algorithm based on the log-likelihood maximization, I extract the best-performing model including three Omitted Effects: those of real wages, inflation, and income. The additional effects represent a meaningful enhancement of the model's ability to adapt to data and, plausibly, forecast future economic trends.

Notably, the impulse response functions of the new estimated model reveal two key results: a positive response of consumption and income following a cost-push shock, and a near-zero consumption response to a government spending shock. These encouraging findings indicate that the RANK model may overlook certain economic mechanisms in which inflation plays a central role, thus warranting further investigation.

References

Andrade, Philippe et al. (2019). “Forward guidance and heterogeneous beliefs”. In: *American Economic Journal: Macroeconomics* 11.3, pp. 1–29.

Auerbach, Alan J and Yuriy Gorodnichenko (2012). “Measuring the output responses to fiscal policy”. In: *American Economic Journal: Economic Policy* 4.2, pp. 1–27.

Baxter, Marianne and Robert G King (1993). “Fiscal policy in general equilibrium”. In: *The American Economic Review*, pp. 315–334.

Bernardini, Marco and Gert Peersman (2018). “Private debt overhang and the government spending multiplier: Evidence for the United States”. In: *Journal of Applied Econometrics* 33.4, pp. 485–508.

Bilbiie, Florin O (2008). “Limited asset markets participation, monetary policy and (inverted) aggregate demand logic”. In: *Journal of economic theory* 140.1, pp. 162–196.

— (2019). “Optimal forward guidance”. In: *American Economic Journal: Macroeconomics* 11.4, pp. 310–345.

Bilbiie, Florin O and Roland Straub (2012). “Changes in the output Euler equation and asset markets participation”. In: *Journal of Economic Dynamics and Control* 36.11, pp. 1659–1672.

— (2013). “Asset market participation, monetary policy rules, and the great inflation”. In: *Review of Economics and Statistics* 95.2, pp. 377–392.

Bilbiie, Florin Ovidiu (2018). “Monetary policy and heterogeneity: An analytical framework”. In.

Blanchard, Olivier and Roberto Perotti (2002). “An empirical characterization of the dynamic effects of changes in government spending and taxes on output”. In: *the Quarterly Journal of economics* 117.4, pp. 1329–1368.

Blanchard, Olivier Jean and Charles M Kahn (1980). “The solution of linear difference models under rational expectations”. In: *Econometrica: Journal of the Econometric Society*, pp. 1305–1311.

Calvo, Guillermo A (1983). “Staggered prices in a utility-maximizing framework”. In: *Journal of monetary Economics* 12.3, pp. 383–398.

Campbell, Jeffrey R et al. (2019). “The limits of forward guidance”. In: *Journal of monetary economics* 108, pp. 118–134.

Campbell, John Y and N Gregory Mankiw (1989). “Consumption, income, and interest rates: Reinterpreting the time series evidence”. In: *NBER macroeconomics annual* 4, pp. 185–216.

Carlstrom, Charles T, Timothy S Fuerst, and Matthias Paustian (2015). “Inflation and output in New Keynesian models with a transient interest rate peg”. In: *Journal of Monetary Economics* 76, pp. 230–243.

Christiano, Lawrence, Martin Eichenbaum, and Sergio Rebelo (2011). “When is the government spending multiplier large?” In: *Journal of Political Economy* 119.1, pp. 78–121.

Christiano, Lawrence J, Martin Eichenbaum, and Charles L Evans (2005). “Nominal rigidities and the dynamic effects of a shock to monetary policy”. In: *Journal of political Economy* 113.1, pp. 1–45.

Corsetti, Giancarlo, André Meier, and Gernot J Müller (2012). “Fiscal stimulus with spending reversals”. In: *Review of Economics and Statistics* 94.4, pp. 878–895.

Del Negro, Marco, Marc Giannoni, Christina Patterson, et al. (2012). “The forward guidance puzzle”. In.

Dornbusch, Rudiger and Stanley Fischer (1993). “Moderate inflation”. In: *The World Bank Economic Review* 7.1, pp. 1–44.

Eggertsson, Gauti B et al. (2003). “Zero bound on interest rates and optimal monetary policy”. In: *Brookings papers on economic activity* 2003.1, pp. 139–233.
Eggertsson, Gauti B and Michael Woodford (2003). *Optimal monetary policy in a liquidity trap.*
— (2004). “Policy options in a liquidity trap”. In: *American Economic Review* 94.2, pp. 76–79.
Fernández-Villaverde, Jesús, Juan Francisco Rubio-Ramírez, and Frank Schorfheide (2016). “Solution and estimation methods for DSGE models”. In: *Handbook of macroeconomics*. Vol. 2. Elsevier, pp. 527–724.
Flavin, Marjorie (1984). *Excess sensitivity of consumption to current income: liquidity constraints or myopia?*
Forni, Mario and Luca Gambetti (2010). “Fiscal foresight and the effects of goverment spending”. In.
Gabaix, Xavier (2020). “A behavioral New Keynesian model”. In: *American Economic Review* 110.8, pp. 2271–2327.
Gabaix, Xavier and David Laibson (2017). *Myopia and discounting*. Tech. rep. National bureau of economic research.
Galí, Jordi, J David López-Salido, and Javier Vallés (2007). “Understanding the effects of government spending on consumption”. In: *Journal of the european economic association* 5.1, pp. 227–270.
Hagedorn, Marcus et al. (2019). “Forward guidance”. In: *Journal of Monetary Economics* 102, pp. 1–23.
Hall, Robert E (1978). “Stochastic implications of the life cycle-permanent income hypothesis: theory and evidence”. In: *Journal of political economy* 86.6, pp. 971–987.
Kaplan, Greg, Benjamin Moll, and Giovanni L Violante (2018). “Monetary policy according to HANK”. In: *American Economic Review* 108.3, pp. 697–743.
Kaplan, Greg and Giovanni L Violante (2018). “Microeconomic heterogeneity and macroeconomic shocks”. In: *Journal of Economic Perspectives* 32.3, pp. 167–94.
Korinek, A (2017). *Thoughts on DSGE macroeconomics: matching the moment, but missing the point?” A Just Society” Conference Honoring Joseph Stiglitz’s 50 years of Teaching. SSRN.*
Kydland, Finn E and Edward C Prescott (1982). “Time to build and aggregate fluctuations”. In: *Econometrica: Journal of the Econometric Society*, pp. 1345–1370.
McKay, Alisdair, Emi Nakamura, and Jón Steinsson (2016). “The power of forward guidance revisited”. In: *American Economic Review* 106.10, pp. 3133–3158.
— (2017). “The discounted euler equation: A note”. In: *Economica* 84.336, pp. 820–831.
Nakata, Taisuke et al. (2019). “Attenuating the forward guidance puzzle: Implications for optimal monetary policy”. In: *Journal of Economic Dynamics and Control* 105, pp. 90–106.
Ramey, Valerie A and Matthew D Shapiro (1998). “Costly capital reallocation and the effects of government spending”. In: *Carnegie-Rochester conference series on public policy*. Vol. 48. Elsevier, pp. 145–194.
Ramey, Valerie A and Sarah Zubairy (2018). “Government spending multipliers in good times and in bad: evidence from US historical data”. In: *Journal of Political Economy* 126.2, pp. 850–901.
Sargent, Thomas J and Neil Wallace (1976). “Rational expectations and the theory of economic policy”. In: *Journal of Monetary economics* 2.2, pp. 169–183.
Smets, Frank and Rafael Wouters (2007). “Shocks and frictions in US business cycles: A Bayesian DSGE approach”. In: *American economic review* 97.3, pp. 586–606.
Stiglitz, Joseph E (1984). *Theories of wage rigidity*. Tech. rep. National Bureau of Economic Research.

Stiglitz, Joseph E (2018). “Where modern macroeconomics went wrong”. In: *Oxford Review of Economic Policy* 34.1-2, pp. 70–106.
Svensson, Lars EO (2014). *Forward guidance*. Tech. rep. National Bureau of Economic Research.

Appendices

A Technical Appendix

A.1 Omitted Effect representation of New-Keynesian Models: Euler Equation

This technical appendix provides details related to Chapter 1 of the paper, focusing on the three models studied: TANK, Cognitive Discounting, and HANK. The Euler equations for consumption in these three models will be replicated using a baseline RANK model. The goal is to first present the reference models, followed by the development of the main equations of the RANK model with different types of omitted effects. The development of the model will not be comprehensive but will be limited to what is necessary for this chapter: analyzing the Euler equation for consumption.

A.1.1 TANK as RANK with Income Omitted Effect

We begin by presenting the TANK model. In the formulation proposed by Galí, López-Salido, and Vallés 2007, a basic New Keynesian model incorporates a preliminary form of agent heterogeneity. Specifically, this framework assumes that the economy is populated by two types of agents: Ricardian agents and Hand-to-Mouth agents. The former, commonly referred to in the economic literature as rational agents, maximize their utility subject to an intertemporal budget constraint. These agents decide, in each period, how much to consume and how much labor to supply. Alongside them, Hand-to-Mouth agents do not maximize any utility function but are constrained to consume their entire disposable income in each period.

The key parameter in this model economy is λ, the proportion of Hand-to-Mouth agents in the population.

The consumption of Ricardian agents follows from the constrained optimization process described below.

$$max_{C_t^r, N_t^r} U_t = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{r\,1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta} \right\} \tag{55}$$

$$s.t.\;\; P_t C_t^r + B_t = W_t N_t^r + B_{t-1}(1+i_{t-1}) + D_t + T_t. \tag{56}$$

The maximization process brings to the consumption Euler Equation:

$$c_t^r = E_t(c_{t+1}^r) - \frac{1}{\sigma}(i_t - E_t(\pi_{t+1})) \tag{57}$$

On the other hand, Hand to Mouth agents consume all their disposable income in every period such that:

$$C_t^h = W_t N_t, \tag{58}$$

that in log-linear form is written:

$$c_t^h = w_t + n_t. \tag{59}$$

We abstract from government spending and taxation as long as this does not affect our main objective. In linear form we write the consumption deviations from steady state of Hand to Mouth agents this way:

$$c_t^h = w_t + n_t. \tag{60}$$

The model considers that aggregate consumption is a weighted average of the consumption of the two agents:

$$C_t = (1-\lambda)C_t^r + \lambda C_t^h \tag{61}$$

Assuming that the steady state level of consumption of the two type of agents is the same, in deviations from the steady state form, the aggregate function reads:

$$c_t = (1-\lambda)(E_t(c_{t+1}) - \frac{1}{\sigma}(i_t - E_t(\pi_{t+1}))) + \lambda(w_t + n_t) \tag{62}$$

Knowing that trought the supply side equations (including the wage shedule) $w_t + n_t = \chi y_t$, and using the market clearing condition $c_t = y_t$, we can derive the aggregate IS function:

$$y_t = E_t(y_{t+1}) - \frac{1-\lambda}{1-\lambda\chi} r_t. \tag{63}$$

Where $r_t = i_t - \pi_{t+1}$.

To compare the TANK IS with the RANK with Income Omitted Effects, we need to develop the model from the optimization of the representative agent.

RANK with Income Omitted Effect

The representative agent maximizes the following Out-Of-Control Utility function under the usual budget constraint.

$$max_{C_t,N_t} U_t = E_t \sum_{k=0}^{\infty} \beta^k \left(\frac{C_t^{1-\sigma}}{1-\sigma} \mathbf{Y_t^\gamma} - \frac{N^{1+\eta}}{1+\eta} \right) \tag{64}$$

$$s.t.\ \ P_t C_t^r + B_t = W_t N_t^r + B_{t-1}(1 + i_{t-1}) + D_t + T_t. \tag{65}$$

That brings to the following linear equation:

$$y_t = E_t(y_{t+1}) - \frac{1}{1-\gamma} r_t \tag{66}$$

for which, comparing the coefficients of the 2 IS functions, we conclude that:

$$\frac{1-\lambda}{1-\lambda\chi} = \frac{1}{1-\gamma}, \tag{67}$$

so that:

$$\gamma = \frac{\lambda(\chi - 1)}{(1-\lambda)}. \tag{68}$$

A.1.2 Cognitive Discounting as RANK with Price Omitted Effect

This appendix outlines a microfoundation for the concept of *cognitive discounting* as reported in Gabaix 2020.

To illustrate the idea, consider a simplified scenario at time $t = 0$, where the agent simulates the next-period value X_1. In reality, $X_1 = G_X(X_0, \epsilon_1)$, but the agent receives a noisy signal Y_1 defined as:

$$Y_1 = \begin{cases} X_1 & \text{with probability } q, \\ X_1' & \text{with probability } 1 - q, \end{cases}$$

where X_1' is an i.i.d. draw from the unconditional distribution of X_1. This mechanism captures disruptions in the agent's reasoning process. Normalizing the unconditional mean of X_1 to zero, the perceived value is given by:

$$\hat{X}_1(y_1) = qy_1,$$

and the average perceived value, conditional on X_1, is:

$$\hat{X}_1(X_1) = q^2 X_1.$$

Defining $m = q^2$, we obtain:

$$\hat{X}_1 = mG_X(X_0, \epsilon_1).$$

The representative agent perceives the future state as $X_{t+1} = mG_X(X_t, \epsilon_{t+1})$ and maximizes the expected utility:

$$U = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right],$$

subject to consumption and production constraints. Aggregate production, in the absence of price frictions, is given by $c_t = \epsilon_t N_t$.

The behavioral agent views the future differently from the rational agent. Linearizing the state evolution, we have:

$$\hat{X}_{t+1} = m(X_t + \epsilon_{t+1}),$$

and the subjective expectations at time t are:

$$\mathbb{E}_t^{\text{BR}}[X_{t+k}] = m^k \mathbb{E}_t[X_{t+k}],$$

where the dampening increases with the horizon k. This reflects a *global cognitive discounting*, where the agent perceives distant future events less accurately compared to a rational agent, embedding the discount factor m^k in all perceived variables. The Consumption Euler equation under cognitive discounting takes the following fomr:

$$c_t = ME_t(c_{t+1}) - \frac{1}{\sigma} r_t. \tag{69}$$

Where M is the cognitive discounting parameters defined between 0 and 1.

RANK with Price Omitted Effect

To represent an Euler equation of the type derived by Gabaix within a RANK model, we need to incorporate the price of the homogeneous good into the Out-of-Control Utility function.

$$max_{C_t,N_t} U_t = E_t \sum_{k=0}^{\infty} \beta^k \left(\frac{C_t^{1-\sigma}}{1-\sigma} \mathbf{P_t^\gamma} - \frac{N^{1+\eta}}{1+\eta} \right) \tag{70}$$

$$s.t.\ P_t C_t^r + B_t = W_t N_t^r + B_{t-1}(1+i_{t-1}) + D_t + T_t. \tag{71}$$

That in turn brings to the linearized IS curve:

$$y_t = (1-\delta\chi)E_t(y_{t+1}) - \frac{1}{\sigma}r_t. \tag{72}$$

Now, comparing the two IS functions we get:

$$(1-\delta\chi) = \mathbf{M}, \tag{73}$$

so that:

$$\delta = \frac{1-\mathbf{M}}{\chi}. \tag{74}$$

A.1.3 a-HANK as RANK with Income and Price Omitted Effects

This section outlines the flows of agents across two islands, H and S, and their implications for resource pooling and welfare maximization. Households on each island migrate as follows:

- A proportion $(1-h)\lambda$ of households leave island H and participate in the following period, while the rest λh remain.
- Similarly, $(1-s)(1-\lambda)$ migrate from island S to H.

At the beginning of each period, the family head pools resources on each island, ensuring symmetric consumption and saving decisions for all households on that island. Denoting the per capita real bonds at the beginning of period $t+1$ as B_{t+1}^j (where $j = S, H$), and the real values after pooling as Z_{t+1}^j, the flows of bonds follow these relationships:

$$B_{t+1}^S = sZ_{t+1}^S + (1-s)Z_{t+1}^H,$$
$$B_{t+1}^H = (1-h)Z_{t+1}^S + hZ_{t+1}^H.$$

$$W(B_t^S, B_t^H) = \max\left\{C_t^S, C_t^H, Z_{t+1}^S, Z_{t+1}^H\right\}\left\{(1-\lambda)U(C_t^S) + \lambda U(C_t^H) + \beta E_t\left[W(B_{t+1}^S, B_{t+1}^H)\right]\right\},$$

subject to:

1. Bond flows' laws of motion:
$$Z_{t+1}^S, Z_{t+1}^H \geq 0,$$
2. Budget constraints:
$$C_t^S + Z_{t+1}^S = Y_t^S + \frac{1+i_{t-1}}{1+\pi_t}B_t^S,$$
$$C_t^H + Z_{t+1}^H = Y_t^H + \frac{1+i_{t-1}}{1+\pi_t}B_t^H.$$

The optimization yields complementary slackness conditions for Z_{t+1}^S and Z_{t+1}^H, and the corresponding Euler equations connect consumption, savings, and expected future utility.

Approximating around a symmetric steady state, the *aggregate Euler-IS equation* relates aggregate consumption and inequality. Individual consumption depends on aggregate income as:

$$c_t^H = \chi y_t, \quad c_t^S = \frac{1-\lambda\chi}{1-\lambda} y_t,$$

where χ depends on fiscal redistribution $(1 - \tau_D/\lambda)$ and labor market characteristics (ϕ).
Equilibrium income inequality is cyclical and defined as:

$$\gamma_t = y_t^S - y_t^H = \frac{(1-\chi)}{1-\lambda} y_t,$$

where:

- Inequality is **procyclical** $(\partial\gamma/\partial y > 0)$ if $\chi < 1$,
- Inequality is **countercyclical** $(\partial\gamma/\partial y < 0)$ if $\chi > 1$.

This model highlights the interaction between income distribution and aggregate dynamics under cyclical conditions.

$$c_t^S = sE_t c_{t+1}^S + (1-s)E_t c_{t+1}^H - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1}\right).$$

Solving and rearranging the term we get the IS curve:

$$y_t = \left(1 - (1-\chi)\frac{(1-s)}{(1-\lambda\chi)}\right) E_t(y_{t+1}) - \frac{1-\lambda}{1-\lambda\chi} r_t. \tag{75}$$

RANK with Income and Price Omitted Effects

$$U_t = E_t \sum_{k=0}^{\infty} \beta^k \left(\frac{C_t^{1-\sigma}}{1-\sigma} \mathbf{Y_t^\gamma P_t^\delta} \right). \tag{76}$$

It brings us to:

$$y_t = \left(\frac{(1-\gamma-\delta\chi)}{(1-\gamma)} \right) E_t(y_{t+1}) - \frac{1}{1-\gamma} r_t. \tag{77}$$

Now comparing the coefficients of the two models it is easy to conclude that:

$$\gamma = \frac{\lambda(\chi-1)}{(1-\lambda)} \tag{78}$$

and

$$\delta = \frac{(1-\chi)(1-s)}{(1-\lambda)\chi}. \tag{79}$$

A.2 Estimation Algorithm for Omitted Effects Selection

Here, we introduce an additional criterion for selecting variables that may be relevant to the model. The criterion proposed here is grounded in empirical analysis and the estimation of the Omitted Effects themselves. Specifically, it involves the use of an iterative estimation algorithm that identifies, for a given model, which among a limited set of admissible unforeseen effects maximize the model's fit to the data.

The term "admissible effects" is used because model estimation is subject to certain constraints, such as the number of shocks and the number of observed variables. Starting from a suitably calibrated baseline model, the iterative algorithm outputs a list of variables for which an Omitted Effect appears significant, as its inclusion improves the model's fit to the data. The algorithm also provides the estimated coefficients associated with these effects.

The algorithm is employed extensively in chapter 5 to identify Omitted Effects related to the RANK model. It is important to note that the algorithm is highly flexible and can be adapted to the needs of the economist, as well as the estimation methodology, which may differ from the one employed here. In our case, I will use Bayesian estimation, but other methodologies are adaptable to the process described here.

Let $\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$ be the set of endogenous and exogenous variables of the DSGE model. Let $\mathcal{P}(\mathcal{V})$ be the power set of $\mathcal{V}$, representing all possible combinations of variables (including the empty set). Let $\mathbb{I}(\mathbf{v})$ be the Information Criterion value for a given combination of variables $\mathbf{v} \subseteq \mathcal{V}$.

1. Define an initial minimum $\mathbb{I}$ value, $\mathbb{I}_{\min} = -\infty$. Define the initial optimal combination, $\mathbf{v}_{\max} = \emptyset$.

2. For each combination of variables $\mathbf{v} \in \mathcal{P}(\mathcal{V})$, compute $\mathbb{I}(\mathbf{v})$. If $\mathbb{I}(\mathbf{v}) < \mathbb{I}_{\max}$, update:

$$\mathbb{I}_{\max} = \mathbb{I}(\mathbf{v})$$

$$\mathbf{v}_{\max} = \mathbf{v}$$

3. The optimal combination of endogenous variables is $\mathbf{v}_{\max}$ with the corresponding $\mathbb{I}_{\max}$.

Formally, the algorithm can be described as follows:

1. Define the set of endogenous variables:

$$\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$$

2. Define storage variables for the minimum I and the optimal combination:

$$\mathbb{I}_{\max} = \infty$$

$$\mathbf{v}_{\max} = \emptyset$$

3. Iterate through all combinations $\mathbf{v} \subseteq \mathcal{V}$:

$$\forall \mathbf{v} \in \mathcal{P}(\mathcal{V}), \quad \text{compute} \quad \mathbb{I}(\mathbf{v})$$

$$\text{if} \quad \mathbb{I}(\mathbf{v}) < \mathbb{I}_{\max} \quad \text{then} \quad (\mathbb{I}_{\max} = \mathbb{I}(\mathbf{v}),\ \mathbf{v}_{\max} = \mathbf{v})$$

4. Return the optimal combination[11]:

$$\mathbf{v}_{\max}$$

The optimal combination $v_{\max}$ represents the set of variables included in the model whose Omitted Effects have been found to be critical in improving the model's fit to the data. Each variable, being associated with an estimated coefficient, allows the algorithm to also provide the estimated value.

11

Initialization:	$\mathrm{AIC}_{\min} \leftarrow \infty$
	$\mathbf{v}_{\min} \leftarrow \emptyset$
Iteration:	for $\mathbf{v} \in \mathcal{P}(\mathcal{V})$ do
	$\quad \mathrm{AIC}_{\mathbf{v}} \leftarrow$ compute $\mathrm{AIC}(\mathbf{v})$
	$\quad$ if $\mathrm{AIC}_{\mathbf{v}} < \mathrm{AIC}_{\min}$ then
	$\qquad \mathrm{AIC}_{\min} \leftarrow \mathrm{AIC}_{\mathbf{v}}$
	$\qquad \mathbf{v}_{\min} \leftarrow \mathbf{v}$
Output:	return $\mathbf{v}_{\min}, \mathrm{AIC}_{\min}$

B Appendix

Table 5: Estimated Coefficients of Omitted Effects in RANK

Omitted Effects	Estimated Coefficients	Log-Likelihood
w, π, y	-1.24, 0.48, 0.59	2455.11
c, y, r	-3.20, 0.74, 1.39	2454.98
c, g, r	-2.46, 0.74, 1.39	2454.98
g, y, r	3.20, -2.46, 1.39	2454.98
g, w, r	0.67, -0.78, 0.52	2454.51
c, r	-2.47, 1.37	2453.22
g, π, y, r	3.13, 0.24, -2.40, 1.36	2452.78
c, g, π, r	-2.40, 0.73, 0.24, 1.36	2452.78
g, w, π, a	0.52, -1.36, 0.75, 0.39	2452.78
c, w, π, y	1.96, -1.20, -0.29, 0.65	2452.78
w, π, a, y	-1.24, 0.49, 0.01, 0.59	2452.78
c, g, π, a	-2.78, 1.02, 1.47, -4.29	2452.78
g, w, π, r	0.66, -0.76, 0.19, 0.51	2452.78
g, π, a, y	3.81, 1.47, -4.29, -2.78	2452.78
g, w, π, y	-0.01, -1.23, 0.48, 0.60	2452.78
w, π, y, r	-1.06, 0.29, 0.62, 0.18	2452.78
g, a, y, r	2.44, -0.84, -1.62, 1.10	2452.70
c, g, a, r	-1.62, 0.82, -0.84, 1.10	2452.70
g, w, a, r	0.77, -0.49, -0.78, 0.56	2452.70
w, a, y, r	-0.71, -0.75, 0.74, 0.32	2452.70
c, g, π, a, r	-1.70, 0.80, 0.16, -0.70, 1.12	2450.44
g, w, π, a, y	0.70, -1.40, 0.84, 0.52, -0.20	2450.44
c, w, π, a, y	-1.21, -1.52, 1.09, 0.88, 0.43	2450.44
c, g, w, π, a	-0.72, 0.44, -1.51, 1.06, 0.85	2450.44
c, g, w, a, r	-0.81, 0.79, -0.24, -0.81, 0.83	2450.35
c, g, a	-3.84, 1.10, -4.82	2449.67
g, a, y	4.94, -4.82, -3.84	2449.67
c, g, w, a	-1.55, 0.94, -0.41, -3.13	2447.33
g, w, π	0.59, -1.40, 0.40	2443.70
w, π, a	-1.37, 0.99, 0.56	2442.42
c, g, w	0.39, 0.60, -1.29	2441.26
g, w, y	0.21, -1.29, 0.39	2441.26
g, w, a	0.56, -1.37, 0.21	2441.26
w, π, a, r	-1.49, 1.18, 0.39, -0.19	2440.35
g, w	0.59, -1.39	2439.54
g, w, a, y	0.78, -1.42, 0.34, -0.24	2438.92
w, π, r	-0.67, 0.46, 0.53	2437.85
g, π, a, r	0.87, 1.23, -1.99, 0.50	2434.22
g, π, a	0.87, 2.06, -2.76	2429.47
g, a, r	0.92, -1.84, 0.71	2426.83
π, a, r	1.29, -1.90, 0.48	2426.09

Omitted Effects	**Estimated Coefficients**	**Log-Likelihood**
π, a, y, r	1.16, -1.84, -0.15, 0.53	2424.43
π, a	2.09, -2.78	2423.18
π, a, y	2.00, -2.95, -0.27	2422.16
w, π	-1.45, 0.49	2421.36
a, y, r	-1.59, -0.36, 0.76	2419.55
π, y, r	1.66, -0.89, 1.01	2418.84
c, a, y	0.88, -1.31, -0.31	2417.83
c, g, π	1.23, 0.76, -0.46	2417.32
g, a	0.31, -0.86	2416.16
w, a	-1.01, 1.12	2415.11

RECENT PUBLICATIONS BY *CEIS Tor Vergata*

A Foundation for Universalisation in Games
Enrico Mattia Salonia
CEIS *Research Paper*, 603 June 2025

On the Estimation of Climate Normals and Anomalies
Tommaso Proietti and Alessandro Giovannelli
CEIS *Research Paper*, 602 June 2025

Meritocracy as an End and as a Means
Enrico Mattia Salonia
CEIS *Research Paper*, 601 May 2025

Nudging Households' Sustainable Investments: Results from a Pilot Lab-in-the-field Experiment In Italy
Beatrice Bertelli, Marianna Brunetti, Costanza Torricelli and Mariangela Zoli
CEIS *Research Paper*, 600 May 2025

Identifying Belief-dependent Preferences
Enrico Mattia Salonia
CEIS *Research Paper*, 599 May 2025

Should I Share or Should I Not? On the Sharing of Information on Past Performance in Procurement
Gian Luigi Albano, Walter Ferrarese, Alberto Iozzi and Roberto Pezzuto
CEIS *Research Paper*, 598 April 2025

When the Going Gets Tough: the Impact of Health Shocks on Divorce
Javier Adrián López Artero, Anna Sanz-de-Galdeano and Daniela Vuri
CEIS *Research Paper*, 597 April 2025

A Stochastic Volatility Approximation for a Tick-By-Tick Price Model with Mean-Field Interaction
Paolo Dai and Paolo Pigato
CEIS *Research Paper*, 596 April 2025

Credit Markets, Corporate Governance and Growth
Emanuele Brancati, Paolo E. Giordani, Maurizio Iacopetta and Raoul Minetti
CEIS *Research Paper*, 595 March 2025

Using Multiple Tools to Enhance Competition in Public Procurement
Clara Calini, Alessandra Catenazzo and Elisabetta Iossa
CEIS *Research Paper*, 594 February 2025

DISTRIBUTION

Our publications are available online at www.ceistorvergata.it

DISCLAIMER

COPYRIGHT

MEDIA INQUIRIES AND INFORMATION

For media inquiries, please contact Barbara Piazzi at +39 06 72595652/01 or by e-mail at piazzi@ceis.uniroma2.it. Our web site, www.ceistorvergata.it, contains more information about Center's events, publications, and staff.

DEVELOPMENT AND SUPPORT

For information about contributing to CEIS Tor Vergata, please contact at +39 06 72595601 or by e-mail at segr.ceis@economia.uniroma2.it