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Green Ambiguity

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Abstract

Agents may be unsure about the productive potential of green technology and of the non-polluting sector due to imprecise information or misguiding news. I study the impact of this deep uncertainty in the context of the transition to a low-carbon economy in a dynamic stochastic general equilibrium model with polluting and green sectors and agents who, due to their ambiguity aversion, take decisions under pessimistic expectations about the future productivity of the latter sector. In the short term, losses of confidence can shift the balance of the economy in favor of investment in the polluting sector and lead to an increase in emissions. Coupling environmental tax and green subsidy can partially counteract this imbalance when the long-run forecast of agents ends up realizing, while also avoiding delays in the green transition. A dynamic version of the policy mix is also able to mitigate the short-term effects of drops in confidence.

JEL classification: Q55; E32.
Keywords: Business Cycle; Ambiguity; E-DSGE.

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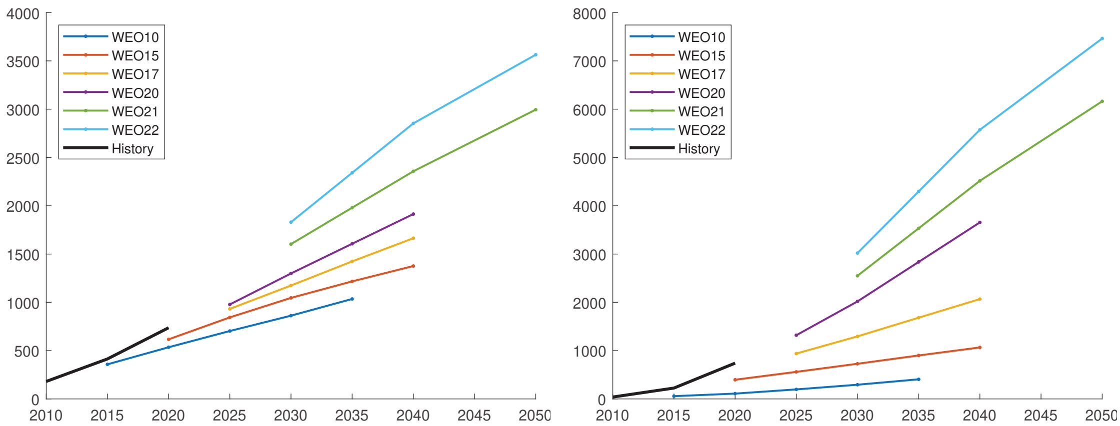
1 Introduction

The disastrous effects of climate change are making increasingly evident the need for drastic reductions in anthropogenic emissions of greenhouse gases (GHGs). As extreme weather events become more frequent and their aftermaths more dramatic, a call for action towards maintaining the average increase in global temperature below 2°C above pre-industrial levels is ever more pressing. Nevertheless, as stated by both the Intergovernmental Panel on Climate Change (IPCC) in its Sixth Assessment Report and the International Energy Agency in its assessment on Energy Technology Perspectives (IEA, 2023), reliance on clean technologies must increase dramatically to reach a net-zero emission target by 2050. In this sense, imprecise information and misguiding or mistrust-inducing news may impair the low-carbon transition if they affect the perception and confidence of agents in the outcomes that the green sector can achieve.

Figure 1: Electrical Capacity (GW)



(a) Wind (b) Solar PV

Note: For a more homogeneous comparison, the analysis starts from 2010 as this is the first year in which the IEA produces three different scenarios for projections. Data are taken from the "New Policies Scenario", where the Agency takes into account proposals that have been made public over the past year but not yet officially adopted. Only a selection of years is displayed. Source: IEA (2022).

Several studies have revealed how the International Energy Agency itself, whose World Energy Outlook (WEO) annual report is often used as a basis for policy-making on renewable and conventional energy, is not always able to correctly predict the potential of some technologies over time. In particular, Metayer et al. (2015) show that IEA's projections for solar photovoltaics (PV) and wind energy, together with forecasts for the capacity (maximum power output) of most other renewable sources of energy, up to the 2014 WEO report, have been consistently underestimated. Figure 1 extends the analysis to later years and confirms the findings: in comparison to the available historical data points, IEA regularly undervalues electrical capacity from wind and solar sources even when revising upwards its estimates in each annual report. In fact, the graphs also highlight a quite consistent pessimistic outlook that the Agency corrects every year, as is especially evident with the projections from the latest reports.

A thorough analysis stressing the difficulty in forecasting the evolution of technological progress in the context of the energy transition is also offered by Meng et al. (2021). As technological trajectories are complex and cannot be fully characterized by any single indicator, the authors focus on the cost per unit of energy as one of the most informative indicators for the assessment of energy technology progress. In this context, they evaluate the performance of several forecasting techniques, such as in reduced-form model-based and expert elicitation methods, displaying the divergence in their results. The various approaches are shown to differ in their uncertainty ranges and, albeit model-based forecasting methods generally outperform elicitation ones, they all underestimate technological progress in almost all selected technologies. In particular, the assessments by the experts reveal a consistent pessimistic bias with respect to realized costs.

These analyses highlight the complexity of assessing the future potential of clean technologies even when they are more mature, as is the case for wind and solar energy. The argument is even more compelling for less established technologies, where the possibility of breakthroughs could push realizations of productivity higher than forecast. Eventually, whenever public reports and information shared by experts are imprecise, also agents' long-run perception may end up being biased. Moreover, in the shorter run, mistrust-inducing news may interact with a distorted forecast and aggravate the lack of confidence revolving around the productive potential of the non-polluting sector.[1] Situations in which probability assignment about the occurrence of some events, like, for instance, the actual realizations of green productivity is neither obvious nor unanimously agreed on, can be framed in the context of "ambiguity".[2] Agents averse to this kind of deep uncertainty find themselves willing to be robust to the worst case happening, consistently with the kind of pessimistic bias highlighted by the evidence above.

The theme of ambiguity in relation to climate policy is not new to macroeconomic

[1]Take as an example the news about Donald Trump's stance on climate and energy legislation upon election (Smyth and Williams, 2023).

[2]This concept, which, in the words of Knight (1921), overlaps with the broad definition of "uncertainty", is to be distinguished from "risk" (and generally from the current understanding of "uncertainty") where instead the reference is to circumstances in which the probabilities (either subjective or objective) of events' occurrence are known.

literature but is mainly cast in terms of decision theory. In the context of the estimation of climate sensitivity (i.e., the temperature change in response to increased atmospheric carbon dioxide concentration), Millner et al. (2013) highlight the existence of a plurality of complex scientific models that often do not agree with one another. Given the insufficient quality of our knowledge about the climate system and the consequent difficulty in assigning probabilities to combine and compare the various estimates, the authors extend the framework by Klibanoff et al. (2005) to study how ambiguous knowledge and aversion to ambiguity affect the welfare analysis of climate change abatement policy. In the same spirit, Manski et al. (2021) explore another approach to use climate simulations in integrated assessment and avoid simple or weighted averaging of the outputs of multimodel ensembles. To account for this "deep uncertainty" (or ambiguity), the authors propose a min-max regret decision criterion and, based on this, develop a theoretical framework for cost-benefit analysis of climate policy. Berger and Marinacci (2020) instead investigate the issue of ambiguity in terms of uncertainty about which model actually governs states' realizations and review recent models of choices under uncertainty to show how different concepts and methods of economic decision theory can be directly useful for problems in environmental economics.
Among the decision criteria discussed by the aforementioned authors, the present research exploits the "multiple priors" approach proposed by Gilboa and Schmeidler (1989) and its intertemporal version axiomatized by Epstein and Schneider (2003). However, while the literature analyzed so far takes a higher-level stance, this paper reconsiders the issue of ambiguity within a single model following the method by Ilut and Schneider (2014), who evaluate the relevance of shocks to confidence (modeled as changes in ambiguity) about future total factor productivity (TFP) in a New Keynesian business cycle model.

In this work, agents instead lack the confidence to assign probabilities to the future realizations of productivity in the non-polluting sector. To address this issue, I develop a real business cycle (RBC) model whose economy consists of two sectors following Acemoglu et al. (2012): a "dirty" and a "green" sector. Specifically, an environmental module is introduced to describe how production in the former sector generates emissions that affect the overall economy. On the other side of the system, it is assumed that agents perceive the green productivity process as ambiguous: this is a parsimonious way to take into account that expectations about the potential of the sector might diverge from the actual path due to lack of confidence. In line with this formalization, households are averse to ambiguity and take their decisions under pessimistic expectations about the future productivity of the green sector. This ultimately implies a shift of the economy towards investment in the dirty sector. The dynamic analysis shows that, even when there is no change in the fundamentals of the economy, short-term drops in confidence can lead to a rise in emissions over the cycle and hinder the low-carbon conversion of the system. Introducing a dynamic policy strategy that employs the revenues from an environmental tax to subsidize the price of the green input is able to partially reinstate the balance of the system. The implementation of a static version of the scheme is instead helpful in counteracting the inefficiency associated with the long-run forecast of agents in case it ends up realizing, while also making the worst-case transition less costly.

Other works have adopted the same approach in business cycle frameworks. Michelacci and Paciello (2019) study the effects of monetary policy announcements when agents face ambiguity about the monetary authority's ability to commit. In this context, the effect of the announcement is attenuated because only wealthy households act as if it will be fully implemented due to the potential wealth losses from future inflation. As a result, the economy responds as if aggregate net wealth falls. Masolo and Monti (2020) highlight the wedges between long-run inflation expectations, trend inflation, and the inflation target driven by ambiguity about the behavior of the policymaker. When ambiguity-averse agents face Knightian uncertainty about the conduct of monetary policy, long-run inflation expectations do not correspond to the inflation target. From a more normative perspective, they show how, in the presence of ambiguity, a simple monetary policy rule that tracks movements in the natural rate of interest can be improved upon by one that tracks a higher interest rate (lower in the vicinity of the ZLB). Ilut and Saijo (2021) analyze cyclical properties when heterogeneous firms face Knightian uncertainty about their profitability and learn it through production. In this setup, they show how the feedback between uncertainty and economic activity endogenously generates co-movement driven by demand shocks, amplified and hump-shaped dynamics, countercyclical correlated wedges in the equilibrium conditions for labor, risk-free and risky assets, and countercyclical firm-level and aggregate dispersion of forecasts. Other works in similar frameworks include: Altug et al. (2020) who examine cyclical dynamics of a Real Business Cycle model with ambiguity-averse consumers and investment irreversibility using the smooth ambiguity model by Klibanoff et al. (2005, 2009); Carbonari and Maurici (2023) who study inequality in an economy with heterogeneous agents; Michelacci and Paciello (2024) who show that, under Knightian uncertainty, households base their actions on beliefs that are negatively distorted by their wishes (the more so the greater the uncertainty they face), reconciling this with evidence from UK households.
To the best of my knowledge, this is the first work that applies this method to treat

ambiguity in the context of an environmental dynamic stochastic general equilibrium (E-DSGE) model in order to study its implications for the green transition.

This paper is related to a growing body of literature on environmental DSGE models, whose first contributions are those of Fischer and Springborn (2011), Heutel (2012), Angelopoulos et al. (2013), and Dissou and Karnizova 2016, who study pollution policies and the optimal responses to the business cycle in environmental variants of the baseline RBC model.[3] A non-exhaustive selection of recent works extending the framework to clean and dirty sectors and inputs include: Argentiero et al. (2017, 2018), who provide an assessment of the effectiveness of policy strategies for renewable energy sources in different countries; Busato et al. (2023), who study how interaction between economic decision-making and environmental awareness affects U.S. business cycle; Guinea et al. (2024), who investigate the impact of anticipated news shocks on renewable and fossil energy use in the U.S. economy. Notably, the work by Campiglio et al. (2024) develops a dynamic model in which beliefs on future carbon prices matter for firms' decisions to invest in either high or low-carbon capital. In this context, weak commitment to mitigation policies can imply a loss of credibility and eventually lead to a failure of the transition.

This paper also broadly encompasses a rapidly expanding literature on Integrated Assessment Models (IAMs), which, following Nordhaus (1993) and starting from the pioneering work by Golosov et al. (2014), uses growth models to study climate policy. A few examples of contributions in the area include: Hassler and Krusell (2012), Rezai and der Ploeg (2016), Barrage (2019), Dietz and Venmans (2019), Hambel et al. (2021), van den Bremer and van der Ploeg (2021), van der Ploeg and Rezai (2021), Traeger (2023)[4]. Notably, Hambel et al. (2024) develop a global economy where consumption goods are produced by both a green and a carbon-intensive sector to analyze asset pricing and climate policy. Closer to the subject of the present work, Traeger (2014) shows the importance of uncertainty and ambiguity aversion in the evaluation of the costs and benefits of policies, like optimal mitigation plans, while Barnett et al. (2020) address preference-based concerns about ambiguity and model misspecification.

The remainder of the paper is organized as follows. Section 2 presents the utility-based theoretical framework in which the analysis is conducted. Section 3 analyzes a reduced version of the model to investigate analytically the impact of ambiguity and optimal taxation. Section 4 describes the calibration of the structural parameters of

[3]For an early discussion on this topic, see Bowen and Stern (2010), while for a review of the literature and discussion on the policy implications, see Annicchiarico et al. (2021). On emission dynamics over the business cycle, see instead Doda (2014) and Klarl (2020).

[4]See Metcalf and Stock (2017) for a review and assessment of IAMs for the U.S.

the model. Section 5 presents a long-run analysis of optimal policies along the lines of Section 3 and investigates the implementation of a green transition. Section 6 studies the short-term dynamics of the system in response to a drop in confidence and explores the role of dynamic policies. Section 7 concludes.

2 The model economy

In the spirit of Acemoglu et al. (2012), the economy is divided into two sides. Firms in the polluting or dirty sector use production inputs that generate emissions that negatively affect the whole economy. On the other hand, agents perceive the process describing the evolution of productivity in the green sector as ambiguous. The inputs from the two intermediate sides of the economy are eventually aggregated into a final good. Ambiguity-averse households take into account the ambiguity around green productivity when making choices about the consumption of the final good and the supply of capital and labor to both sectors.

2.1 Final-good producers

The final good Y_t is produced by a representative firm through a combination of the intermediate inputs from the dirty, Y_t^d, and the green, Y_t^g, sector purchased respectively at price P_t^d and P_t^g according to the aggregate constant elasticity of substitution (CES) production function:

$$Y_t = \left[\nu^{\frac{1}{\rho_Y}} (Y_t^d)^{\frac{\rho_Y - 1}{\rho_Y}} + (1-\nu)^{\frac{1}{\rho_Y}} (Y_t^g)^{\frac{\rho_Y - 1}{\rho_Y}} \right]^{\frac{\rho_Y}{\rho_Y - 1}} , \tag{1}$$

where $\rho_Y > 0$ is the elasticity of substitution between the two sectors and ν the share (or weight) of the dirty sector. When $\rho_Y > 1$, the inputs from the two sectors are (gross) substitutes; when $\rho_Y < 1$, the inputs instead are (gross) complements.[5] In the calibration of the model, it is assumed that some degree of substitutability across sectors is present.[6]

[5]When $\rho_Y \to 1$, the function collapses to a Cobb-Douglas-like aggregator.

[6]The role of this assumption is analyzed throughout the work and especially in the reduced version of the model in the next section.

2.2 Dirty sector

The dirty intermediate good Y_t^d is produced according to the following Cobb-Douglas production function:

$$Y_t^d = [1 - D_t(M_t)]A_t^d(K_t^d)^\alpha(H_t^d)^{1-\alpha} \tag{2}$$

where A_t^d evolves according to the following log-normal process:

$$\log A_{t+1}^d = \rho_{A^d}\log A_t^d + \varepsilon_{A^d,t+1} \tag{3}$$

In the spirit of the DICE model by Nordhaus (2008) and as in Heutel (2012) and Annicchiarico and Di Dio (2015) among others, the production process is assumed to be polluting, and emissions E_t depend on dirty production:

$$E_t = \chi(Y_t^d)^\varepsilon, \tag{4}$$

where $\chi > 0$ is a scale parameter and ε is the sensitivity of emissions to production. Polluting gases accumulate into a stock M_t according to the following law of motion:

$$M_t - \bar{M} = \sum_{s=0}^{t+T}(1 - \delta_M)^s E_{t-s} \tag{5}$$

where $\bar{M}$ denotes the pre-industrial concentration of pollutants, $\delta_M \in (0, 1)$ measures the natural decay rate of greenhouse gases in the atmosphere, while E indicates global emissions. Following Golosov et al. (2014) and Nordhaus (2008), the accumulation of polluting emissions negatively affects production in both sectors through a damage function D_t:

$$1 - D_t(M_t) = \exp\left[-\xi\left(M_t - \bar{M}\right)\right], \tag{6}$$

where $\xi > 0$ is a damage parameter measuring the intensity of the negative environmental externality on production (or, analogously, the fraction of output lost for each extra unit of pollutants). Climate change is then a function of the accumulated stock of emissions rather than of emissions as a flow at any time and it is therefore to be intended as a stock externality.[7]

[7] This is a parsimonious way to introduce the effects of climate change into the model. This is standard practice in several aggregate models where it is implicitly assumed that climate change is a function of the atmospheric stock of greenhouse emissions.

2.3 Green sector

Similarly to the other side of the economy, the green intermediate good Y_t^g is produced according to the following Cobb-Douglas production function:

$$Y_t^g = [1 - D_t(M_t)]A_t^g(K_t^g)^\alpha(H_t^g)^{1-\alpha} \tag{7}$$

which is again affected by the accumulation of polluting gases in the atmosphere. However, productivity in the sector A_t^g follows a process of the kind:

$$\log A_{t+1}^g = \rho_{A^g} \log A_t^g + \mu_t^* + \varepsilon_{A^g,t+1}. \tag{8}$$

The component μ_t^* defines a family of processes that differ in their conditional means, and agents are thought to be unable to assign probabilities on the different possible paths. In particular, following the same formalization as in Ilut and Schneider (2014), the empirical moments of μ_t^* are restricted in such a way that it becomes impossible to disentangle the effects of such component from the shocks sequence $\varepsilon_{A^g,t}$.[8] Hence, agents respond to such deep uncertainty by treating the parameter μ_t^* as ambiguous.
In particular, individuals develop beliefs about the ambiguous component by gathering information from experts' reports, scenarios, and news. The complex and possibly imprecise nature of this type of evidence affects the confidence of agents in their assessment of future green productivity, enabling them only to put bounds around the next-period forecast. The interval defining the belief set of agents is parametrized as $[-a_t, a_t]$, where a_t summarizes perceived ambiguity. Specializing in this way the values that the component μ_t^* can take, Equation (8) can be recast as

$$\log A_{t+1}^g = \rho_{A^g} \log A_t^g + \mu_t + \varepsilon_{A^g,t+1}, \quad \mu_t \in [-a_t, a_t] \tag{9}$$

where a_t is assumed to follow the log-normal process

$$\log a_{t+1} = \rho_a \log a_t + (1 - \rho_a) \log \bar{a} + \varepsilon_{a,t+1} \tag{10}$$

and $\varepsilon_{a,t}$ is a normal i.i.d. innovation. As ambiguity increases, so does the belief set, which implies a decrease in confidence for agents. The persistence in the process can be interpreted as the cumulative effect of news that affects confidence. Agents know the

[8] Each innovation to green productivity is defined as $z_{t+1}^{A^g} \equiv \log A_{t+1}^g - \rho_{A^g} \log A_t^g$. The empirical moments of μ_t^* are assumed to converge in the long run to those of an i.i.d. normal process with mean zero (i.e. $\mu^* = 0$), independent of $\varepsilon_{A^g,t}$ and with variance $\sigma_z^2 - \sigma_{A^g}^2 > 0$. Innovations z^{A^g} then fully resemble realizations of an i.i.d. process with mean zero and variance σ_z^2.

law of motion for a_t and eventually form expectations accounting for it.[9]

2.4 Households

The representative household's utility is a function of consumption C_t and hours worked in the two sectors H_t^k with $k \in \{d, g\}$:

$$U(C_t, H_t^k) = \frac{C_t^{1-\gamma}}{1-\gamma} - \sum_{k \in \{d,g\}} \frac{(H_t^k)^{1+\sigma_H}}{1+\sigma_H} \tag{11}$$

where γ measures the degree of risk aversion and σ_H is the inverse of Frisch labor supply elasticity.

The household is averse to ambiguity, and the lack of confidence in probability assessments is captured by means of recursive multiple priors preferences (Epstein and Schneider, 2003). Denote as $s_t \in S$ the vector referring to the exogenous states and s^t the history up to date t of these variables. Then, a household consumption plan C gives, for every history s^t , the consumption of the final good $C_t(s^t)$ and the amount of hours worked in the two sectors $H_t^k(s^t)$. For a given consumption plan C, lifetime utility is defined recursively by:

$$V_t(C, H^k; s^t) = U(C_t, H_t^k) + \beta \min_{\mu_t \in [-a_t, a_t]} \mathbb{E}_t^{\mu}[V_{t+1}(C, H^k; s^t, s_{t+1})] \tag{12}$$

where $\beta \in (0, 1)$ is the subjective discount factor, while the expectation operator $\mathbb{E}_t^{\mu}$ is used to make explicit the dependence of expected continuation utility on the conditional mean μ_t. When the belief set is a singleton, the framework reduces to the standard expected utility case; when it is not, it reflects the assumption that the agent perceives Knightian uncertainty.[10] In response to this, the household takes a cautious approach to decision-making and acts as if the green productivity process is defined by the worst-case (with associated pessimistic beliefs denoted by $\mathbb{E}_t^{-a}[.]$). Such minimization of the expected continuation utility captures a strict preference for knowing probabilities.

[9] Notice that this representation for the exogenous processes allows to always obtain positive values for a_t, which is consistent with the definition of the belief set, with an interval centered around zero.

[10] When the belief set is instead maximal, i.e. contains all possible prior probabilities, the framework reverts to the standard max-min criterion. Moreover, notice that the max-min expected utility case is, in turn, obtained as a limiting case of the smooth ambiguity model (Klibanoff et al., 2005) when the ambiguity aversion parameter goes to infinity.

The period-by-period budget constraint is:

$$C_t + B_t + \sum_k I_t^k = \sum_k W_t^k H_t^k + \sum_k r_t^k K_t^k \tag{13}$$

Households' sources of funds are earnings from labor $W_t^k H_t^k$ and supply of capital $r_t^k K_t^k$. These funds are allocated to consumption and investment in the two sectors I_t^k. Capital in the two sectors accumulates according to the following equations with adjustment costs:

$$K_{t+1}^k = I_t^k + (1-\delta)K_t^k - \frac{\phi_I}{2}\left(\frac{I_t^k}{K_t^k} - \delta\right)^2 K_t^k \quad \text{for} \;\; k \in \{d, g\} \tag{14}$$

In period t the typical household chooses C_t, H_t^k, K_{t+1}^k with the objective of maximizing (12) subject to (13) and (14).

The full set of equilibrium conditions describing the model economy are summarized in Appendix A.

3 Reduced model

In order to explore more in depth the impact of green ambiguity, this section considers a reduced version of the main model that can be solved in closed form. Production in the two sectors is given by the following technology:

$$Y_t^k = A_t^k H_{t-1}^k \quad \text{for } k \in \{d, g\} \tag{15}$$

Notice that it is now assumed that labor is the only production input, and work exerted at date $t-1$ only bears fruit at a later stage. Possible ways to interpret this are that the production process takes a while to complete or that goods have to be stored for some time before they can be consumed.[11] Moreover, although dirty production is assumed to produce emissions linearly (according to (4) when $\varepsilon = 1$), let us consider for the time being that damage on production is negligible. For tractability, assume further that the final good is derived from a Cobb-Douglas aggregator (as $\rho_Y \to 1$):

$$Y_t = (Y_t^d)^{\nu}(Y_t^g)^{(1-\nu)} \tag{16}$$

[11]It may be helpful to think of the period length as very short. The assumption is needed to make the choice of labor intertemporal and dependent on uncertainty.

Exogenous processes for productivity in the two sectors are also simplified as follows:

$$\log A^d_{t+1} = \varepsilon_{A^d,t+1} \tag{17}$$
$$\log A^g_{t+1} = \mu_t + \varepsilon_{A^g,t+1} \tag{18}$$

namely, both innovations are serially independent and normally distributed with variance respectively $\sigma^2_{A^d}$ and $\sigma^2_{A^g}$ and mean 0 and μ_t. Once again, agents treat the unknown component μ_t as ambiguous, and their one step-ahead set of beliefs at date t is parametrized by a set of means $[-a_t, a_t]$, where a_t evolves persistently according to (10).
The decentralized problem corresponds to that of a social planner who maximizes with the same multiple priors preferences as agents, being robust to the worst-case scenario arising when $\mu_t = -a_t$.
The closed-form solution for log hours h^g worked in the green sectors reads as:

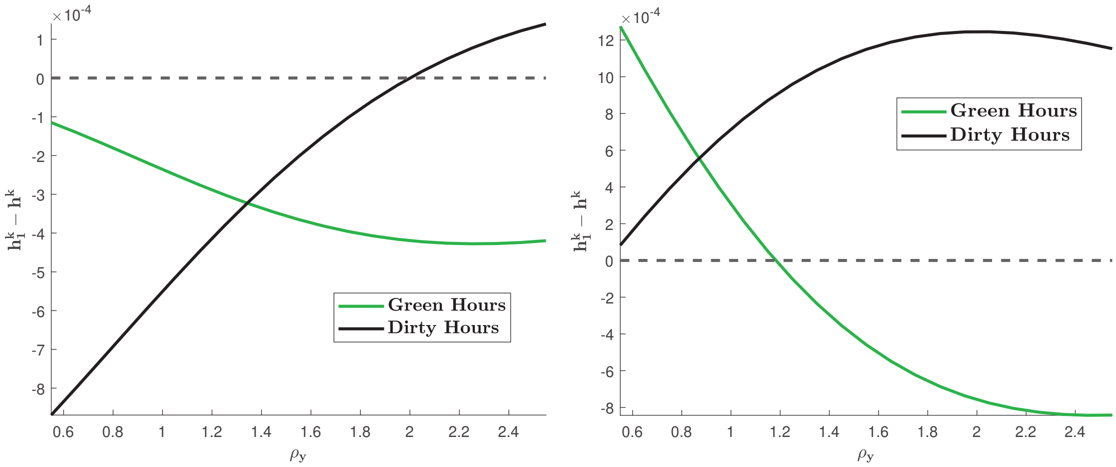
$$h^g_t = \frac{1}{\gamma}\left[(1-\nu)(1-\gamma)(-a_t) - \bar{h}g + \Sigma\right] \tag{19}$$

where Σ is a compound term depending on the variances of both processes as described in Appendix B.1. Let us focus on the effects of a positive deviation of ambiguity from its mean, namely a decrease in the confidence of agents about green technology. Notice that evaluating the reaction on impact does not allow to distinguish between cases in which worst-case expectations realize and those in which they do not.[12] The magnitude of the impact of the ambiguity shock (intended as the change from the steady state in the amount of hours worked at time $t = 1$) depends on the share of green to total production: specifically, it increases as the green input gains more weight in the final production, namely as ν gets closer to zero. Moreover, the effect of ambiguity shrinks as the coefficient of risk aversion γ approaches one (hence utility tends to logarithmic form) from the left, while it grows in absolute size as γ increases away from 1. In the absence of an explicit term describing ambiguity aversion, it is risk aversion that plays a pivotal role in determining the sign of the impact of the shock: whenever $\gamma < 1$ and thus the substitution effect prevails, optimal choice and higher willingness to trade off future with current consumption induce a reduction in hours worked in the non-polluting sector. As the negative effect of ambiguity on the return from labor in the green sector propagates to overall production and there is no substitution between sectors, the shock also impacts

[12] In this analysis, the type of formalization chosen to model ambiguity effectively induces an equivalence of the response on impact of the system between a loss of confidence and a standard negative shock to productivity. Section 6 instead makes a point on the differences in the system's response under the two concepts in the full model economy.

negatively the other side of the economy.[13] On the other hand, when $\gamma > 1$, the income effect prevails, and a higher desire to avoid large changes in consumption induces an increase in hours as the certainty equivalent of overall future production is lowered by ambiguity.
The analytical results summarized above hold when there is no substitution between the two sectors, as the final product is obtained from a Cobb-Douglas aggregator, while Figure 2 shows the sign of the impact of a positive ambiguity shock when the final good is produced according to a CES function like (1).

Figure 2: Impact of positive change in ambiguity on hours worked



(a) $\gamma < 1$ (b) $\gamma > 1$

Note: The figure shows the sign of the impact (intended as the change from the steady state h^k in the amount of hours worked at time $t = 1$ h_1^k for $k \in \{d, g\}$) against the value of ρ_Y, i.e. the elasticity of substitution between sectors. The magnitude of the impact is to be considered only in qualitative terms. Other calibrated values are: $\beta = 0.99$, $\sigma_h = 1$, $\nu = 0.7$, $\bar{a} = 0.005$.

Numerical results confirm the previous findings: in the limiting case of $\rho_Y \rightarrow 1$, which corresponds to the Cobb-Douglas type of aggregator, the impact for hours worked in both sectors has the same sign: negative when $\gamma < 1$ and positive in the opposite case. However, this does not hold true anymore when a higher degree of substitution between sectors comes into place: as ρ_y increases markedly away from 1, the optimal response of green hours to the shock becomes persistently negative. Notice that when the income effect prevails (see Figure 2b), a low degree of substitution is sufficient to induce a positive impact only in the sector whose technology is not perceived as ambiguous. This is the case because individuals are already willing to work more to avoid changes

[13] Equation (B-9) shows that, net of a constant term, the impact of a shock to ambiguity is analytically the same for hours worked in the dirty sector.

in consumption and a lower incentive is sufficient to make them switch to the other sector. Given standard values for the calibration of the elasticity of substitution between production inputs (around 2 as estimated by Papageorgiou et al. 2017), a positive shock to ambiguity is expected to eventually impact negatively hours worked in the green sector and positively those on the other side of the economy, regardless of risk aversion.

3.1 Optimal policies

Consider now the case of a global flow pollutant generated by the dirty sector that affects directly production in the economy. This implies the following functions:

$$Y_t^k = \left[e^{(-\xi E_t)} \right] A_t^k H_{t-1}^k \quad \text{for } k \in \{d, g\} \tag{20}$$

In this case, differently from what happens in the decentralized scenario, the social planner takes into account the negative externality stemming from the dirty sector. In order to achieve the first-best allocation in the decentralized equilibrium, a Pigouvian tax can be devised to induce internalization of the externality (see Appendix B.2 for details). In particular, the instrument is implemented as an *ad valorem* tax on the value of the dirty input sold to final-good producers.

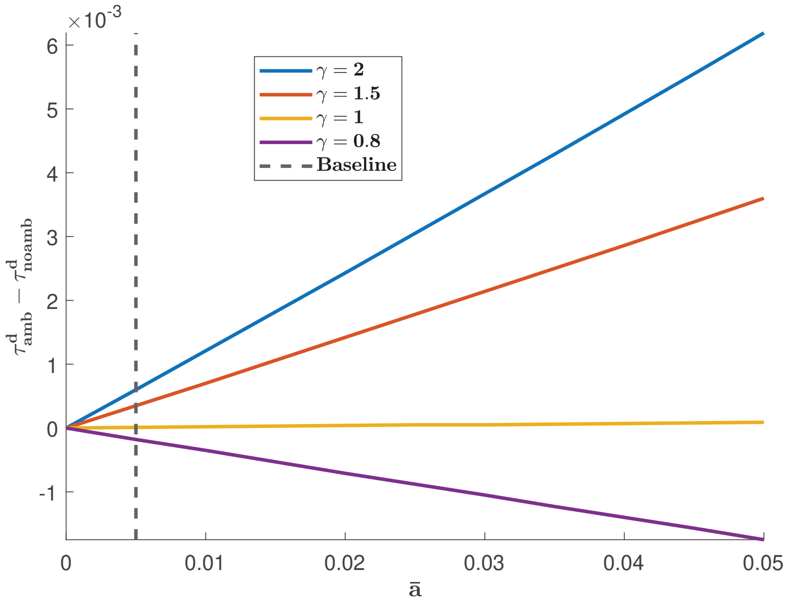
With respect to what would happen in a steady state without ambiguity, the distorted belief of agents introduces a wedge in the definition of the optimal tax, whose size is determined again by the combination of parameters ν and γ:

$$\tau_{amb}^d = e^{-\bar{a}(1-\nu)(1-\gamma)} \tau_{noamb}^d. \tag{21}$$

This additional element describing the direct effect of ambiguity through green productivity is increasing in absolute terms in the share of green to total production and also grows with the coefficient of risk aversion. When $\gamma > 1$, the wedge is increased with respect to the no-ambiguity scenario consistently with the income effect inducing a raise in overall hours worked driven by the dirty sector. Indeed, this causes polluting production to rise to the detriment of the green side of the economy, implying higher emissions and increased environmental damages.

Calibrating output loss from emissions to match the latest projection estimates in the baseline scenario of the DICE model (Barrage and Nordhaus, 2024), the numerical exercises reported in Figure 3 show that an *ad valorem* decentralized environmental tax behaves consistently with the above analytical results. When the CES aggregator collapses to the Cobb-Douglas type (i.e. $\rho_Y \to 1$), risk aversion greater than one implies a higher optimal environmental tax in the distorted steady state with respect to the no-ambiguity scenario. Figure 3 also highlights that the opposite is true when $\gamma < 1$ and the substitution effect prevails. As shown in Appendix B.2, similar results hold when considering the CES case, indicating a lower relevance of this parameter in shaping the optimal constant environmental tax.

Figure 3: Deviation from no-ambiguity optimal environmental tax



Note: The figure shows the p.p. deviation from the no-ambiguity optimal environmental tax as steady-state ambiguity $\bar{a}$ increases for different values of risk aversion γ and $\rho_Y \to 1$.

In order to further evaluate the benefits of imposing a certain type of policy, let us determine a benchmark measure of welfare. Specifically, since the rational expectation case is nested in the multiple prior environment adopted to model ambiguity-averse individuals, the cost of ambiguity is defined as the welfare loss deriving from the distortion in the beliefs of agents. The measure is determined as the portion of consumption that would equate the welfare in the no-ambiguity steady state to the welfare in the case where ambiguity is instead present. In practice, let us define the welfare associated with the time-invariant equilibrium as $U^0 = u^0(C, H^d, H^g)/(1-\beta)$, with $u^0(C, H^d, H^g)$ being the no-ambiguity period-by-period utility. Defining the welfare in the distorted steady state as $\underline{U}^{-a} = u^{-a}(\underline{C}, \underline{H}^d, \underline{H}^g)/(1-\beta)$, the cost ς is computed as (see Appendix B.3

for details):

$$u^0(C(1-\varsigma), H^d, H^g)/(1-\beta) = \underline{U}^{-a}$$

Table 1 reports the welfare results of different numerical exercises from the reduced model with a CES aggregator like in (1). The calibration of the substitution between inputs from the two sides of the system ensures that the impact of ambiguity on economic activity has different signs across sectors.

Table 1: Welfare, ambiguity and policies

	No Ambiguity	Ambiguity
No policy	-204.5533	-206.1677
Environmental Tax	-204.4435 $\tau^d = 0.0578$	-206.0560 $\tau^d = 0.0582$
Policy Mix	-204.4681 $\tau^d = 0.0202$, $\tau^g = -0.0416$	-206.0835 $\tau^d = 0.0188$, $\tau^g = -0.0428$

Note: Damages are computed in reference to the latest projection estimates of the DICE model (Barrage and Nordhaus, 2024). The baseline case is calibrated to match an output loss of 4.4% corresponding to a temperature increase of 3.8 degrees in the current policies scenario. Other calibrated values are: $\gamma = 2$, $\rho_Y = 2$, $\beta = 0.99$, $\sigma_h = 1$, $\nu = 0.7$, $\bar{a} = 0.005$.

The first line illustrates the steady-state welfare when no ambiguity is present against the one that realizes when the distortion perceived by agents materializes. The welfare cost brought about by the interaction of the misperception of the productive potential of the green sector and damages from pollution, in this case, amounts to 1.5463%. Since ambiguity pushes economic activity in the dirty sector up to the detriment of the non-polluting side of the economy, production, and welfare are more negatively affected by the distortion in beliefs when emissions are harmful.

The second and third lines of Table 1 instead take the stance of a regulator who, following the same logic of the derivation of condition (21), computes the welfare-maximizing values for policy instruments under the same ambiguity-averse preferences of agents with the aim of tackling the environmental externality. The first exercise implements a simple environmental *ad valorem* tax τ^d on the value of the dirty input sold to final-good producers, while the third scenario explores the effects of devising a policy mix in which a direct subsidy to the price of the green input is totally financed by the revenues from the environmental tax, namely $\tau^g P^g Y^g = \tau^d P^d Y^d$.

In the first experiment, the environmental tax turns out to be optimal as it depresses economic activity on the polluting side of the economy and favors green production through substitution between sectors. This mechanism correctly tackles the environmental externality, but also partly attenuates the asymmetry induced by the ambiguity perceived on the productive potential of the clean sector. Indeed, implementing the instrument under ambiguity allows to obtain higher welfare gains than what would happen without the distortion, as also confirmed by the lower welfare cost of 1.5168%

The second exercise confirms the desirability of inducing a shift of the economy towards non-polluting activities. In fact, a policy mix is overall beneficial in terms of welfare, even though it does not offer additional gains when implemented under ambiguity (with a welfare cost of 1.5479%). However, the scheme provides the non-secondary advantage of requiring the imposition of a lower environmental tax. This is due to the fact that implementing a green subsidy gives greater incentives with respect to a tax alone to use more of the inputs from the green sector. Welfare gains in this sense are limited by the higher weight of the dirty sector, whose penalization impacts more strongly total production and consumption, and this is especially true when the distortion aggravates the imbalance of the system towards the polluting side.

Eventually, even if different policy schemes are shown to be beneficial in welfare terms, they still seem far from fully and adequately tackling the issue of ambiguity. This is especially true because such instruments try to reinstate the balance of the system and correct the implications of ambiguity without addressing the cause of the distortion.

4 Calibration

The model economy is calibrated to world data and time is measured in quarters. The parameters of the model are partitioned into two categories: a standard macroeconomic part and parameters associated with the environmental externality and the relation between sectors. Table 2 summarizes the calibration.

Regarding standard parameters related to the RBC part of the model, the discount factor β is set to a value consistent with a real interest rate of 4% per year. The depreciation rate of capital δ is set to 0.025, the production parameter α to 0.33, relative risk aversion γ to 2 and the inverse of Frisch elasticity σ_H to 1. This gives the possibility to match some first moments to average values of global macroeconomic aggregates from the World Bank over the time-span 1987-2019, like, for instance, the share of consumption and capital formation over world GDP.

Following the strategy by Ilut and Schneider (2014), parameters describing ambiguity are calibrated on the basis of the cross-sectional dispersion for quarterly forecasts for real

Table 2: Calibrated parameters and steady-state ratios

	Description	Value
Steady state ratios and values		
C/Y	Private consumption	0.7650
$(I^g + I^d)/Y$	Total investment	0.2350
M	Projected stock of carbon	1650.537
RBC parameters		
β	Discount factor	0.99
γ	Risk aversion	2
δ	Depreciation rate of capital	0.025
α	Capital share	0.33
ϕ_I	Adj. costs parameter	0.1
ϕ_H	Adj. costs parameter	0.1
σ_H	Inverse of Frisch elasticity	1
A^d	Dirty sector productivity	1
$\bar{a}$	Ambiguity	0.005
A^g	Green sector productivity	0.9512
Environmental parameters		
$\bar{M}$	Pre-industrial concentration of carbon	581
δ_M	Decay rate of greenhouse gases	0.0021
χ	Emission intensity parameter (implied)	1.6253
ξ	Damage function parameter	4.2072e-05
ϵ	Emissions sensitivity parameter	0.7
ρ_Y	Elasticity of substitution dirty-green	2
ν	Share of dirty firms	0.7
Shocks		
ρ_{A^d}	Autocorrelation Dirty Prod. shock	0.90
σ_{A^d}	Std. Deviation Dirty Prod. shock	0.01
ρ_{A^g}	Autocorrelation Green Prod. shock	0.90
σ_{A^g}	Std. Deviation Green Prod. shock	0.01
ρ_a	Autocorrelation Ambiguity shock	0.90
σ_a	Std. Deviation Ambiguity shock	0.16

US GDP level from the Survey of Professional Forecasters (SPF) from 1985Q1 to 2019Q4. In the model, the counterpart of the measure is defined according to the agents' belief set about next-period green technology given the relation between green productivity innovation and log GDP level (see Appendix C.1 for details).[14] The steady-state level

[14]Even though the calibration is at world level, this strategy seems a sensible option to obtain a lower-

of ambiguity is eventually set to match the average dispersion over the period, which implies a value of A^g under ambiguity that is still close and comparable to A^d, which is in turn fixed to 1 for convenience. A^g would take value 1 if there were no dispersion. This approach also gives the possibility to choose a value for the standard deviation of the ambiguity shock that allows to mimic the volatility the dispersion measure. As business cycles can be generated also by shocks to productivity in each sector, parameters defining correlations and standard deviations are selected in combination to match the order of magnitude of a selection of second moments for the main macroeconomic aggregates observed for the biggest world economy in 1985Q1-2019Q4.[15] In particular, with the values reported in Table 2, the model is able to closely match the observed standard deviation of the GDP and approximately reproduce higher volatility of investments over the period under consideration.

To calibrate the environmental block of the model, the main reference is the DICE model by Nordhaus (2018) and its most recent estimates (Barrage and Nordhaus, 2024). The stock of atmospheric concentration of carbon is matched to 1650 gigatons of carbon (GtC), the approximated value projected in the current policy scenario in the DICE model in 2100. M is then computed according to the fact that the pre-industrial atmospheric concentration of carbon, $\bar{M}$, is about 581 GtC. The quarterly decay rate of greenhouse gases δ_M is fixed at 0.003 to reflect a half-life of carbon in the atmosphere of nearly 83 years, consistently with Reilly and Richards (1993). The overall level of emissions immediately follows from (5). The emission intensity parameter χ is then implied. Using the latest projection estimates of the DICE model on the fraction of output lost because of environmental damages in 2100, which is around 0.044, the damage function parameter ξ is finally computed. The first rationale behind this calibration strategy is to capture higher damages from emissions so as to give more weight numerically to the negative externality of pollution. When factoring in all types of damages related to climate change affecting productivity, a higher figure also appears more in line with recent experience. The second reason is to be consistent with the concept of a worst-case steady state: if the pessimistic expectations are realized, the economy experiences a state with higher emissions. With such a calibration strategy, this is consistent with a scenario in which the concentration of carbon in the atmosphere in the longer run ends up being higher than what would happen without ambiguity.[16]

bound benchmark for the level of ambiguity. Moreover, the measure provides a lower bound to green technological ambiguity also because non-polluting production makes up only a fraction of total output and its forecast is likely to be subject to deeper uncertainty.

[15]The model is solved using Dynare 5.5 up to a second-order approximation.

[16]Specifically, all environmental parameters are derived in a scenario without ambiguity. Implied

Regarding the relation between sectors, the elasticity of substitution between the two ρ_Y is set to 2 relying on empirical estimates by Papageorgiou et al. (2017), the sensitivity of emissions parameter ϵ to 0.7 following Heutel (2012) and the share of green firms $1-\nu$ to an approximate value of 0.3 consistent with the share of electricity production from renewable sources at world level.

5 Full model: long-run analysis

This section replicates the static analysis carried out in Section 3, with the intent of describing some long-term features of the model economy. Specifically, what follows investigates the implications in terms of policies of the worst-case steady state in which the expectations of the agents are realized. Firstly, the optimal environmental tax is characterized analytically to understand the interactions with ambiguity. Secondly, numerical results for the welfare-maximizing policy mix are reported to illustrate the sensitivity with respect to increasing levels of steady-state misperception of agents. The discussion is then also carried over to an economy transitioning from a high to a low-emission state. In this context, the analysis explores the welfare cost of a green transition when agents' worst-case expected outcome is the one that ends up occurring.

5.1 Optimal policies

Introducing capital as the new intertemporal variable of choice in the model and changing the specification of damages from emissions complicates previous considerations. In order to better understand the interplay between ambiguity and optimal policy when the pessimistic expectations of agents are realized in the long run, the optimal Pigouvian tax to induce internalization of the pollution externality is derived analytically. To do so, a social planner with the same utility as the agents is assumed, thus the result can only be considered a first best outcome under the ambiguity-averse preferences described in (12).[17] The optimal instrument is devised as an *ad valorem* tax $\tau^{*,d}$ on the value of the dirty input sold to final-good producers that allows to internalize through P^d the

values are then kept fixed when the distortion is present so that the estimates for the stock of carbon and emissions are higher in the worst-case projections.

[17]The standard first best outcome derived under classic preferences is anyway comparable as the rational expectation case is nested in the multiple prior framework. Notice that ambiguity does not appear explicitly, so the analytical outcomes are apparently the same.

wedge coming out of the social planner solution (see Appendix C.3 for details):

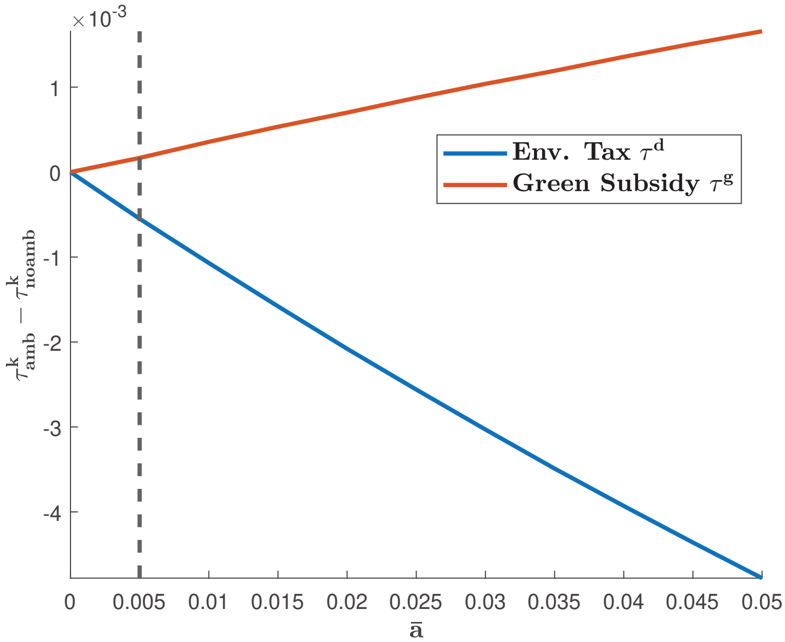
$$\tau^{*,d} = \frac{\tilde{P}^M \chi \epsilon (Y^d)^{\epsilon-1}}{\tilde{P}^d} \tag{22}$$

with $\tilde{P}^d = \nu^{\frac{1}{\rho_Y}} \left(\frac{Y^d}{Y}\right)^{-\frac{1}{\rho_Y}} + \tilde{P}^M \chi \epsilon (Y^d)^{\epsilon-1}$, where $\tilde{P}^M = \frac{\xi\left(\tilde{P}^g Y^g + \tilde{P}^d Y^d\right)}{1-\beta(1-\delta_M)}$ is the discounted marginal damage from emissions on total production value. The optimal instrument is increasing in the marginal damage of polluting production $\chi \epsilon (Y^d)^{\epsilon-1}$ and decreasing in the marginal product of dirty input $\nu^{\frac{1}{\rho_Y}} \left(\frac{Y^d}{Y}\right)^{-\frac{1}{\rho_Y}}$, namely the tax is less penalizing when the dirty sector production has higher impact on final output. As ambiguity induces higher Y^d, the two elements both decrease and affect the tax in opposite directions, so that the optimal scheme and the distortion in beliefs have a less straightforward relationship with respect to what is shown in Section 3. However, what seems to matter the most in numerical terms for the shape of the carbon instrument is the discounted marginal damage from emissions. The tax is increasing in $\tilde{P}^M$, which in turn raises with the total value of production in the economy. Quantitatively, ambiguity depresses capital investment in the whole economy, afflicting almost exclusively those in the green sector. While it is true, as before, that the income effect induces a rise in overall hours worked, the incidence of the decrease in total investment weighs more heavily on the whole production value. Therefore, with respect to the no-ambiguity scenario, the worst-case steady state induces a lower level of green production. This in turn implies a decrease in overall production as capital investments are depressed also on the polluting side of the economy. Marginal damages from emissions are thus eventually reduced as total production value ends up being lower. Ultimately, this implies that the optimal constant tax need not be raised as worst-case steady-state distortion increases.

Along the lines of the analysis in Section 3.1, it is again possible to show the welfare desirability of correcting the imbalance induced by ambiguity by means of a policy mix, where the regulator introduces the welfare-maximizing *ad valorem* tax τ^d on the value of the dirty input sold to final-good producers and exhausts the revenues to finance a subsidy τ^g to the green sector on a balanced budget. Since ambiguity pushes economic activity in the dirty sector up to the detriment of the non-polluting side of the economy, the environmental externality is aggravated. The presence of a distortion therefore gives an additional rationale to study a scheme that, albeit suboptimal, provides a direct incentive to move towards the less polluting side of the economy, thus trying to address jointly the issues coming from emissions and ambiguity.

Figure 4 reports how the tax and subsidy under increasing levels of steady-state

Figure 4: Deviation from zero-ambiguity policy mix



Note: The figure shows the p.p. deviation from the zero-ambiguity optimal environmental tax and implied subsidy ($\tau^k_{amb} - \tau^k_{noamb}$ for $k \in \{d, g\}$) as steady-state ambiguity $\bar{a}$ increases.

distortion compare with respect to the zero-ambiguity scenario. Consistently with the analytical discussion, the environmental tax now decreases as the distortion in the beliefs of agents accentuates. When polluting production gains more weight due to ambiguity, the regulator finds it optimal to tax it less so as to avoid penalizing excessively total consumption. On the other hand, implementing a policy mix always gives room to support the green sector, and this incentive to shift away from the dirty side of the economy decreases the tax needed to correct the inefficiencies and maximize welfare. Moreover, as a lower burden is imposed on a greater taxable base, revenues can support a higher subsidy to low-carbon activity. Introducing the zero-ambiguity policy in a distorted scenario would then imply a suboptimally higher tax. Nevertheless, commitment to a balanced budget scheme would also induce an increased subsidy for the green sector, thus providing even a stronger stimulus aligned with environmental objectives.

As reported by the figures in Appendix C.4, the relation between the zero-ambiguity optimal mix and the scheme under distortion is consistent across different sensitivity exercises. While there appears to be little difference between the two cases when the tax raises following increases on damages from emissions or the weight of green production, greater elasticity of substitution makes the shift toward polluting activities more severe, thus inducing a less aggressive scheme under distortion. Moreover, as ρ_Y and $1-\nu$ gets higher, the marginal product of the dirty input decreases and this raises the optimal environmental tax. However, this does not translate into an increase of the implied subsidy, as technology is already providing enough incentives to transition towards the greener side of the economy.

5.2 Green transition

The implications of the worst-case scenario, that is the one in which pessimistic expectations of ambiguity-averse agents are realized, can also be appreciated in the context of a green transition of the economy to a new state in which emissions are reduced. In particular, assume that a mitigation plan is announced in the first period, is fully credible, and is implemented via a linearly increasing environmental *ad valorem* tax set to reach a reduction in emissions of around 20% in the next ten years. As shown in Appendix C.5, the direction of the transition is the expected one: as the tax increases over time, economic activity in the dirty sector is reduced, especially in terms of capital. On the other side of the system, substitution between inputs induces an increase in green output sustained by higher investment and hours worked. Since the weight of the dirty sector on total production is assumed to stay constant over the time horizon under analysis, the strict mitigation approach to limit emissions brings about a fall in total output.[18] In this context, a distorted perception of the productive potential of the green sector can give rise to a slowdown in the transition. With capital investment being almost unaffected, the less pronounced reduction in hours worked is what keeps output from the polluting sector and emissions higher in the worst case. More importantly, this effect is less counteracted by the shift to the greener side of the economy, as agents are less inclined to invest and work in the less productive sector. Eventually, in order to induce the same reduction in emissions as in the no-ambiguity case over the same time horizon under the scenario that follows agents' worst-case expectations, a higher tax or a more stringent mitigation approach should be used. In this sense, Table 3 quantifies in consumption-equivalent units the additional cost in terms of welfare of undertaking a green transition when the pessimistic view of agents materializes. This surplus of costs is derived by comparing the present discounted value of welfare over the time horizon of the transition which arises under the two different scenarios. Imposing the same tax path as the no-ambiguity case under agents' worst-case perception implies low costs, but it fails to reach the same level of emissions at the transition endpoint. In fact, devising

[18]Given the stylized and small nature of the model, the magnitude of the transition is only to be taken as illustrative.

a more aggressive scheme for the environmental instrument to reach the same emission target makes the green transition costlier in terms of welfare.
Implementing a policy mix, as the one discussed in the previous section, can prove to be beneficial in partially reducing the cost of the transition in case the pessimistic expectations of agents were to be realized. Although non-welfare-improving in this context, such a scheme provides support to the non-polluting side of the economy, with higher investment and hours worked in the green sector in the distorted scenario. Even if emission targets still take longer to be met under ambiguity, this also implies that the policy mix eventually brings about a lower impact of the mitigation approach on total production. Table 3 shows numerically how the policy mix scheme induces a lower cost for the transition under ambiguity with respect to just imposing an environmental tax when the aim is to reach the same emissions level at the endpoint as the no-ambiguity scenario.

Table 3: Welfare cost of transition

	Same emission target	Same tax path
Environmental tax	0.42%	0.08%
Policy mix	0.23%	0.08%

Note: The first column imposes the same emissions level at the transition endpoint as a no-ambiguity scenario; the second column implements the same environmental tax at the end of the transition path as the no-ambiguity case. Welfare costs are computed in absolute consumption equivalent units and reported as percentage change from no-ambiguity case.

In conclusion, if the worst-case path envisioned were to materialize, coupling an environmental tax with a green subsidy would appear to be a sensible policy choice. On the one hand, it is able to address the environmental externality aggravated by ambiguity attitudes of agents, while also providing stronger incentives to invest and work in the non-polluting sector. On the other, the mix is even less costly than an environmental tax alone when the economy undertakes a transition towards a state with reduced emissions. The next section will show how such a policy scheme is beneficial even if the pessimistic expectations of ambiguity-averse agents are not realized on average, for instance, when confidence in the productive potential of the green sector falls in the short term.

6 Full model dynamics

This section explores the short-term response of the system to a drop in confidence. This can be conceptualized as cases in which analyses failing to correctly assess the potential of some green technologies are released or any distrust-inducing news spread out. For instance, information coming out of policy-relevant discussions and reports (like, for example, the already mentioned World Energy Outlook by the IEA) might align agents' views with projections that underestimate the potential of green technology. There might also be more general cases in which news may induce an attitude of mistrust towards the future achievements of non-polluting sectors. For example, any announcements of withdrawing from climate pledges or claims to shift away from commitments to support less polluting activities (like during Trump's administration in the U.S.) could push agents into expecting a favoring of carbon-intensive production processes, thus inducing short-term effects akin to the ones presented in what follows. Analytically, the fall in confidence corresponds to a positive shock to perceived ambiguity a_t in Equation (10). In practice, this shock temporarily broadens the belief set of agents, who end up being even less confident about the process from which the realization of productivity in the green sector comes from. Under their worst-case expectations, the shock has a direct effect on productivity in the green sector, which persists for some quarters as news usually clusters over time. Applying the solution method described in Appendix D gives the possibility to analyze the response of the system to a pure shock on expectations in which green productivity does not deteriorate overtime as instead envisioned by agents. Taking equation (8) to have a zero conditional mean as the benchmark case (which would coincide with the perceived law of motion of green productivity under rational expectations), the correction method allows to evaluate the response of the system when agents solve under pessimistic thinking but actual realizations do not follow the expected worst case.[19] The section finally makes a point on the difference between a shock to ambiguity and other comparable shocks.

6.1 Impulse response analysis

As anticipated, the long-run error in the forecast of the productive potential of the green sector opens the door to possible short-term drops in confidence that widen the belief set of agents. As shown in Appendix C.6, a decrease in the productivity of the green sector actually takes place over the first quarters when the pessimistic expectations of agents are realized. In particular, the shape of green productivity reflects the fact that

[19]When applying this method, the ergodic steady state of the linearized model is one in which the variances of the shocks only have an effect on the endogenous variables through ambiguity (or, in the words of Ilut and Schneider (2014), "zero risk steady state"). In other words, simulations after one shock converge back to the steady state that agents choose expecting a deterioration of green productivity that does not realize. This steady state is numerically close to the worst-case scenario, thus the results in the previous sections would stand valid even considering this different specification.

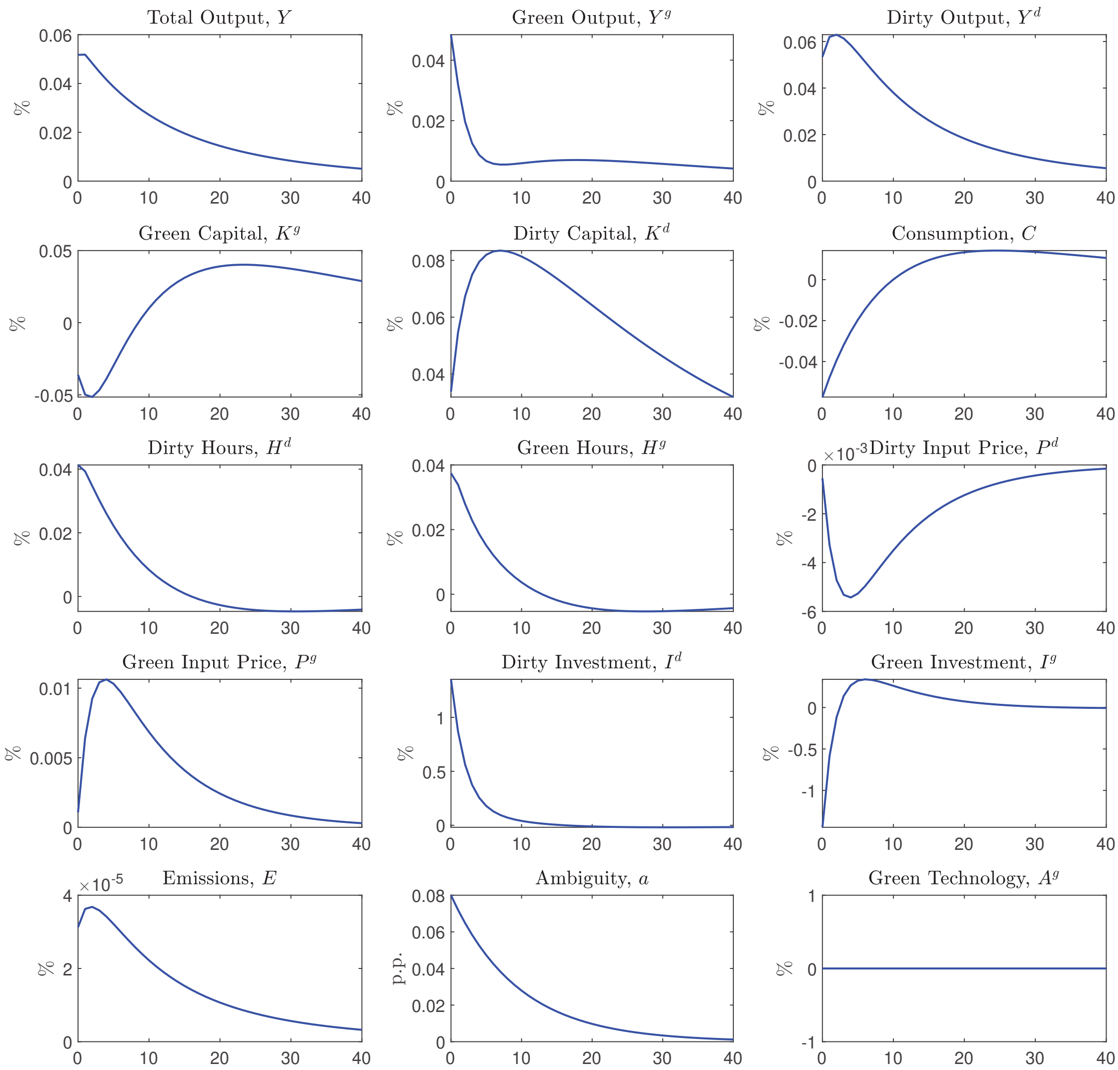
agents expect the worst-case path to deteriorate for a number of quarters as the nature of news and predictions affecting confidence is usually to be clustered in time. The effects on the economy follow the hump-shaped path of marginal productivity of inputs in the non-polluting sector, whose economic activity is consequently depressed over some quarters as firms cut back on capital investment and labor. On the other side of the economy, higher investment and production are sustained by the shift of the demand from final-good producers towards polluting inputs. This partially substitutes for the decrease in economic activity in the green sector, but not as much as to avoid a general decline in consumption and total output, and leads to a rise in emissions.

Figure 5 instead shows the response of the system to a pure shock on expectation, when the loss in confidence does not translate into the realization of low green productivity, contradicting the expectations of agents. Even if the reduction in the productive potential of non-polluting production never comes about and there is no change in the fundamentals of the economy, the positive ambiguity shock makes the balance of the system shift towards the dirty sector.

On impact, Figure 6 shows that the analysis carried out in the reduced model about hours worked now broadly applies to capital investments, as they represent the variables on which an intertemporal choice is made. Pessimistic expectations of agents coupled with the elasticity of substitution between production inputs lead to an increase in investment in the polluting sector to the detriment of the green one over the first quarters. Hours worked in the two sectors instead react positively as individuals care to avoid changes in consumption and precautionary motives push them to work more to increase savings. As this effect is accentuated when the income effect is stronger, the magnitude of the impact on capital is lower when $\gamma > 1$.

Eventually, cautious behavior by agents in the face of a shock that could hamper the potential of green technology favors an increase in economic activity in the dirty sector that lasts over time and is sustained by higher investment in the supposedly more productive capital good. Differently from what happens when a reduction in productivity actually materializes, there is a positive effect on green output too. This is also brought about by the increase in hours worked in the sector deriving from an income effect, however investment in capital is depressed over the first quarters. Overall, the effects of the shock are smaller than when the worst case materializes (see Figure C-6) because realizations for the variables of the system contradict the pessimistic path expected by agents, who find themselves with better outcomes than what envisioned and this brings about a quicker restoration of the balance in the system. This adjustment process also leads to a rebound of green capital investment on a longer horizon, as agents have ex-

Figure 5: Dynamic response to a positive ambiguity shock

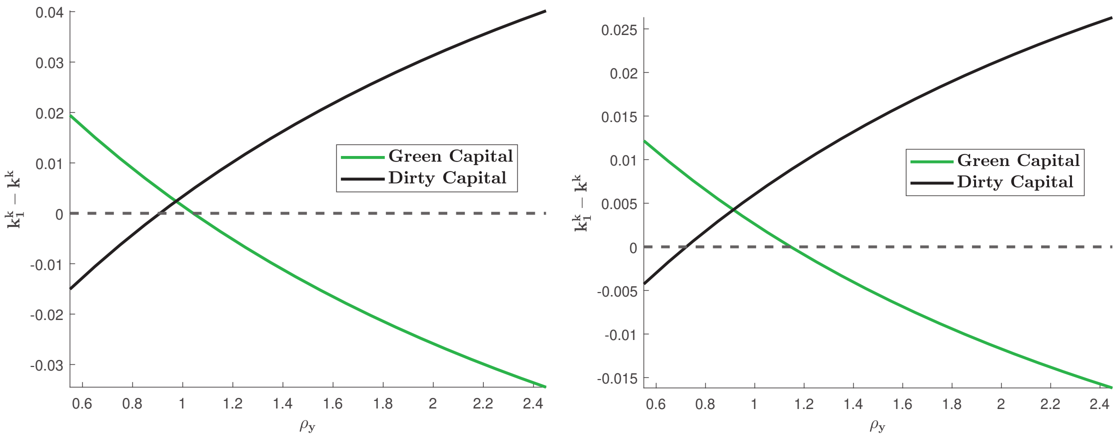


Note: Results are reported over a 40-quarter time horizon as percentage deviations from the no-ambiguity steady state.

perienced higher returns from the green sector while instead expecting a reduction in marginal productivity over the first quarters. Nevertheless, the shock to green ambiguity eventually induces a persistent rise in emissions if pessimistic expectations do not materialize.

This environment allows us to investigate more closely the difference arising under ambiguity and a shock to green productivity. Appendix C.6 reports a comparison between this standard productivity shock and the scenario in which the worst-case expected

Figure 6: Impact of a positive change in ambiguity on capital



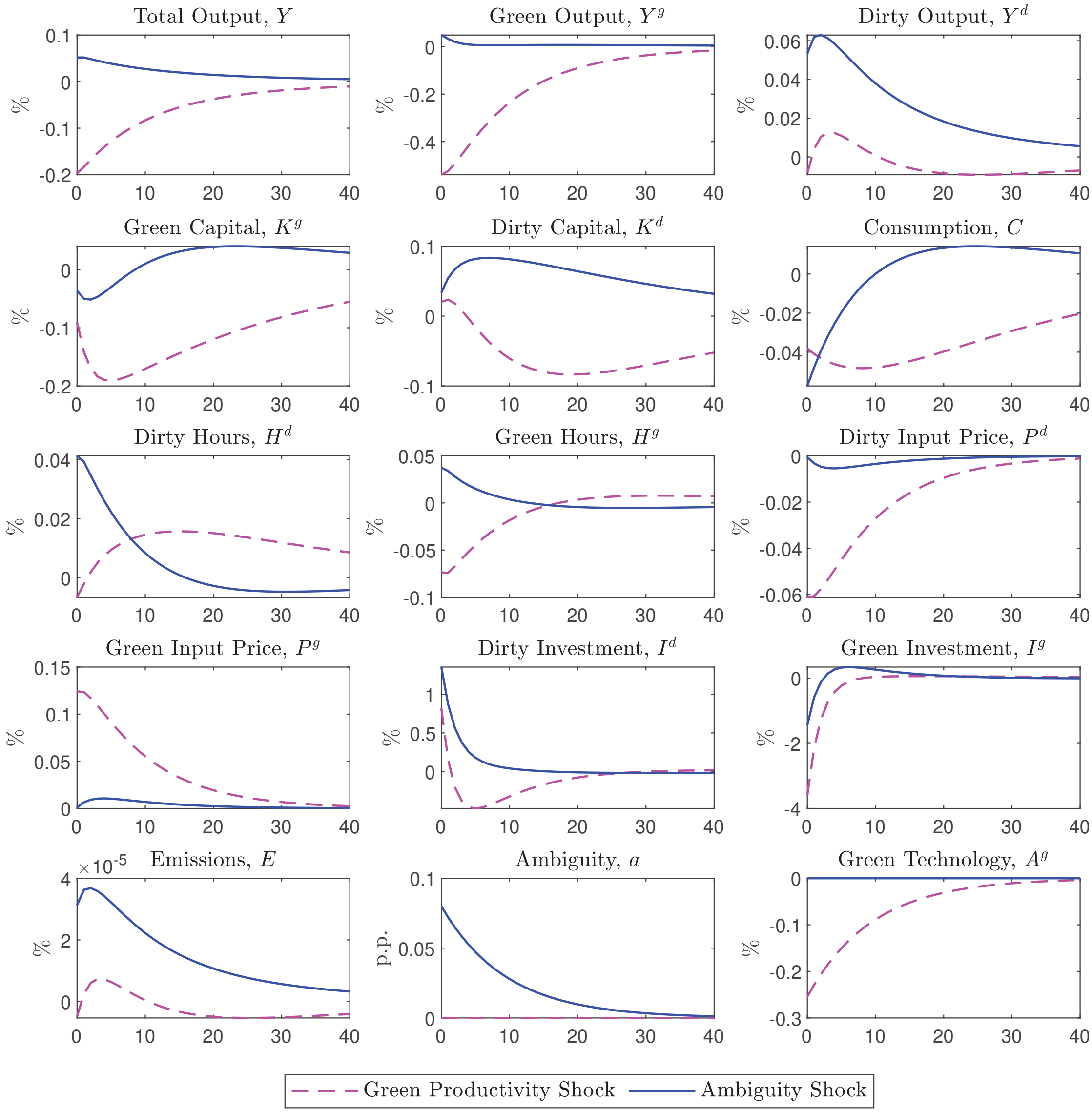
(a) $\gamma < 1$ (b) $\gamma > 1$

Note: The figure shows the sign of the impact (intended as the change from the steady state at time $t = 1$ of log capital k^k for $k \in \{d, g\}$) against the value of ρ_Y, i.e. the elasticity of substitution between sectors. The magnitude of the impact is to be considered in qualitative terms.

by agents after a loss in confidence in the productive potential of the green sector materializes. The economy that experiences a decrease in green productivity is shown to react in a qualitatively close fashion to the case of a realized positive ambiguity shock in terms of the differential impact across sectors. Nevertheless, the economy features some distinct responses that can also be appreciated in Figure 7, where instead, the focus is again on the response of the system in response to a pure loss of confidence. When a standard negative shock to technology in the non-polluting sector is realized, the fall in the marginal productivity of labor and capital induces a reduction for both variables in the non-polluting sector. This brings about a decrease in green output which is, in turn, reflected in the total amount. Overall, a generalized recession is generated. On the polluting side of the economy, capital is brought up by a substitution effect on impact but is soon dragged down by the general slowdown in economic activity. Even if agents end up working more in the dirty sector, the negative effect of reduced investment on output is only partially counteracted. This all results in a generalized recession, but with a lower decline of consumption with respect to the ambiguity shock case.[20] This points to precautionary motives being stronger in the event of a loss of confidence: forecasting a continuous decrease of green productivity over the first quarters, agents are willing to give up more consumption and work more to increase investments in the asset

[20]This comparison can be made as the trough in green technology when pessimistic expectations realize is of the same size as the standard shock to productivity (see Figure C-7).

Figure 7: Dynamic response to positive ambiguity shock v. negative green productivity shock

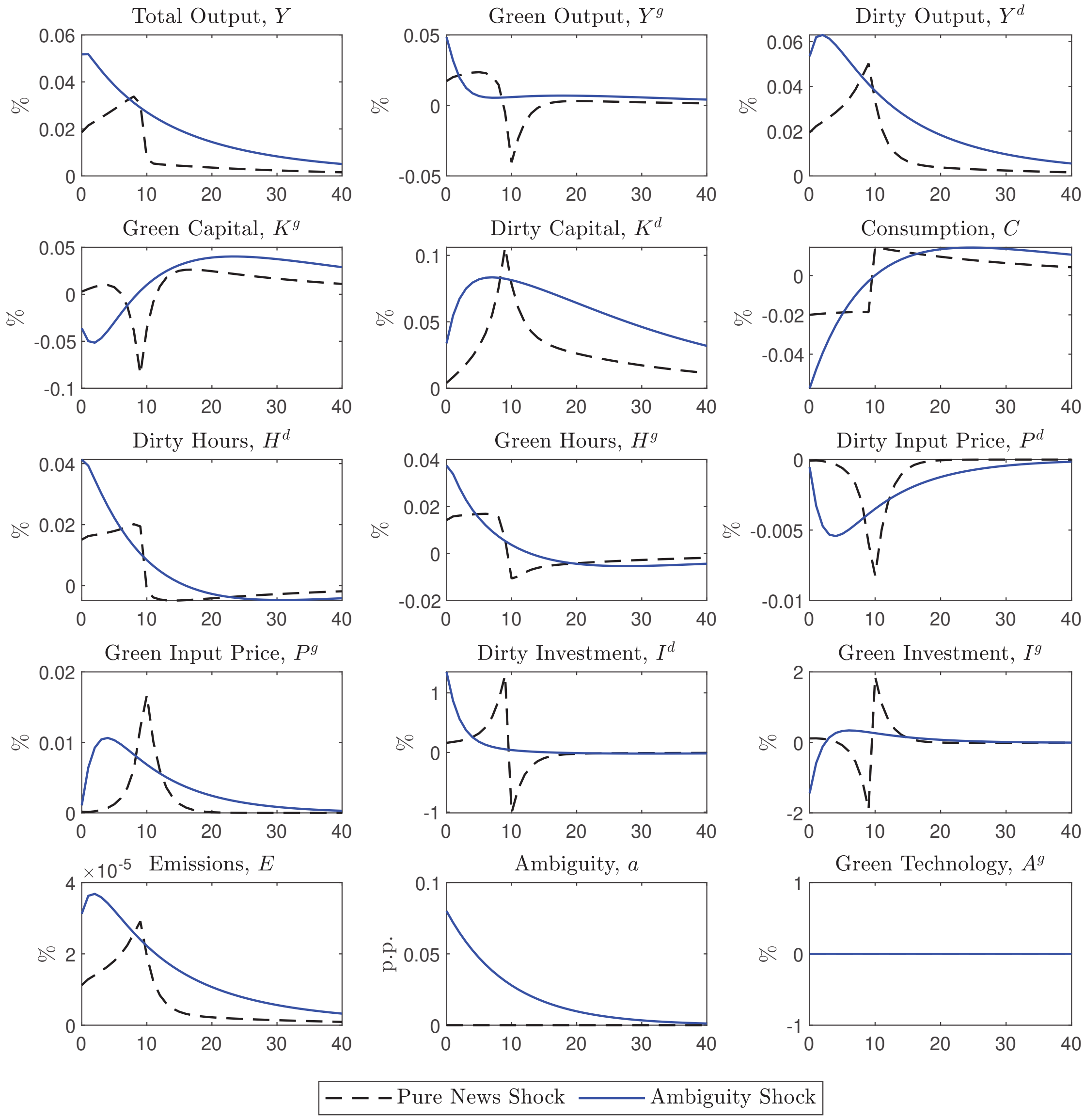


Note: Results are reported over a 40-quarter time horizon as percentage deviations from the no-ambiguity steady state.

that is deemed to be more productive while investing less in the non-polluting sector. However, notice also that negative effects on variables are in general less persistent due to expectations never realizing.

As a last exercise, Figure 8 proposes a comparison between ambiguity and pure news

Figure 8: Dynamic response to positive ambiguity v. pure news shock



Note: Results are reported over a 40-quarter time horizon as percentage deviations from the no-ambiguity steady state.

shock, which can theoretically mimic a similar loss of confidence. In particular, here agents anticipate a shock that will negatively affect productivity in the green sector ten quarters later, but then, when the time comes, such a decrease does not materialize. The reaction of the system is qualitatively comparable to the ambiguity scenario, as also highlighted by Figure C-8 in which the two same-sized realized versions of the shock are reported. Approaching the moment of the shock, agents decrease their capital investment in the sector, which will suffer from the loss in productivity, and the balance of the system tilts in favor of the polluting side of the economy. In terms of hours worked precautionary motives in anticipation of the shock are still at work and push households to work more. However, with respect to the ambiguity case, now the trough is reached at the time when the shock is supposed to occur. Indeed, agents act rationally in anticipation of the green productivity decrease and only become aware of the pure shock nature of the shock when it does not materialize. That is why the system enjoys a sudden recovery towards equilibrium to compensate for the mistaken path undertaken until then. Here resides the main difference with the ambiguity framework, in which, instead, the misjudgment in the assessment is embedded in the way individuals perceive the economy. As they pessimistically solve the model, the actual realizations after a drop in confidence constantly surprise them and shape the response of the system described in this section. Moreover, differently from what would happen under pure news or an uncertainty shock, this setup offers the possibility to link long-run misperceptions on the productive potential of the green sector to short-term inefficiencies due to losses of confidence.

Ultimately, the dynamic analysis indicates that ambiguity poses a challenge to the low-carbon conversion of the economy, especially if implemented in the presence of a lack of confidence in the productive potential of the green sector.

6.2 Dynamic policies

The objective of this last section is to ascertain if variations in ambiguity as the one presented above are detrimental to the economy and investigate whether the introduction of a dynamic policy mix can be beneficial.

Movements in confidence around the productive potential of green technology are shown to be inefficient by evaluating the welfare cost of fluctuations ς generated by shocks to ambiguity. Specifically, this is defined as:

$$\varsigma = 1 - \left[\frac{\mathbb{E}_t\{U\} + \dfrac{{H_{ss}^{d}}^{1+\sigma_H} + {H_{ss}^{g}}^{1+\sigma_H}}{(1+\sigma_H)(1-\beta)}}{\dfrac{C_{ss}^{1-\gamma}}{(1-\gamma)(1-\beta)}} \right]^{\frac{1}{1-\gamma}} \tag{23}$$

where $\mathbb{E}_t\{U\}$ is the theoretical unconditional mean welfare and $C_{ss}, H_{ss}^d, H_{ss}^g$ refer to steady-state variables. In the baseline calibration, this cost turns out to be small (0.0313 b.p.). However, the magnitude of the figure is sensitive to the standard deviation of the ambiguity shocks σ_a, which can be considered as a lower bound in the calibration strategy adopted.

The simulations do not impose any restriction on the directions of changes in ambiguity so there are instances in which negative shocks act as small gains in confidence. However, the nature of the exogenous process ensures that the values of the distortion remain consistent with the specification of the belief set. This implies that there is less room for negative shocks, making positive shocks more relevant, especially as σ_a increases (see Appendix C.7 for a graphical representation).

Building on the discussion in the previous section, the welfare cost of fluctuations is primarily due to inefficient variations in consumption in response to the more influential positive shocks to ambiguity. Precautionary motives drive agents to work more and invest in the dirty asset, fearing a reduction in the productive potential of the green sector. The implementation of a static scheme that penalizes the polluting sector and employs the tax revenues to finance a subsidy to green production is optimal in the sense of maximizing mean welfare induced by shocks to ambiguity.[21] Nevertheless, it is now also possible to devise an environmental tax that responds to variations of emissions along the cycle generated by shocks to ambiguity:

$$\tau_t^d = \left(\frac{E_t}{E}\right)^{\psi^d}. \tag{24}$$

The subsidy to the green sector is, as before, implied by the tax revenues in order to maintain a balanced budget at every point in time. In this context, setting an instrument that penalizes more the polluting sector when emissions are high, with an optimal value of 0.28 for ψ^d, turns out to be optimal in reducing the welfare cost of swings in confidence.[22] As highlighted in Figure C-10, this type of policy allows to mitigate the rise in economic activity in the polluting sector while sustaining capital investment and hours worked in the green one, thus limiting the increase in emissions. On the other hand, note that this type of dynamic policy also eases the burden on dirty production when emissions are lower throughout the cycle.

In conclusion, the analysis carried out throughout the paper confirms that a tax-subsidy scheme, either static or dynamic, is an effective way to counteract the effects of green ambiguity over different time horizons. Nevertheless, this approach seems still

[21] Specifically, the optimal value of τ^d that maximizes $\mathbb{E}_t\{U\}$ is 0.0038.

[22] The dynamic scheme is optimal also when anchored to other variables, for example, to total output with $\psi^d = 0.34$.

far from appropriately addressing the root of the problem. From a policy perspective, the type of intervention able to completely offset the distortion eventually hinges upon the beliefs of agents, which can be shaped by providing more precise and higher-quality information.

7 Conclusions

This paper poses the question of whether the transition towards a low-carbon economy could be hindered by agents mistrusting the productive potential of the non-polluting sector. Given possible imprecise pieces of information and misguiding news, individuals may be skeptical in the assessment of what outcomes can be achieved in the future by clean technologies. Specifically, consistently with their ambiguity-averse preferences, agents entertain pessimistic expectations about the path that productivity in the green sector could follow. Losses of confidence appear to be a non-trivial obstacle in the conversion to a low-carbon economy in the short term. Indeed, the dynamic analysis shows that, even without changes in the fundamentals, the cautious behavior of agents induced by deep uncertainty brings about a tilting of the system in favor of investment in the polluting sector, leading to an increase in emissions. If the pessimistic expectations of agents materialize, the economy ends up in a state characterized by higher production from the dirty sector and, consequently, more emissions. In this context, coupling an environmental tax with a green subsidy is beneficial in counteracting the imbalance induced by ambiguity, with the advantage of providing more incentives to work and invest in the non-polluting sector and avoiding delays in the green transition. A dynamic version of the scheme is instead suited to mitigate the effects produced by variations in confidence that could hinder the ability of the economy to shift away from polluting production. However, such a policy scheme is not enough to fully address the issues at the root of the distortion: providing more precise and higher-quality information is ultimately the type of intervention most suited to offset such green ambiguity.

The paper is a first attempt at analyzing the interaction between an ambiguous perception of green technology and the transition towards a more sustainable economy. In order to isolate the effects induced by ambiguity aversion, the model is kept very stylized. One possible step for future research could be to replicate a similar analysis in a richer framework: on the one hand, an endogenous structure for the evolution of technology could be introduced, possibly with ambiguity reducing as more investments are directed to the green sector; on the other, one could for example allow for ambiguity also in the dirty sector due to possible transition risks and stranded capital. Further research could also implement a similar setup to investigate other interactions with climate issues. For example, it could be interesting to study how ambiguity around the future occurrence of tipping points in environmental damages might influence the design of climate policy. Finally, it might be of interest to revisit the results obtained in this analysis in a different setup, for example by specifying other types of preferences displaying ambiguity aversion or by departing from the representative agent framework to allow for richer dynamics.

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Appendix A: Equilibrium conditions

This appendix describes the equilibrium conditions for the economy, which is characterized by 21 endogenous variables $\{C_t,\, H_t^k,\, \lambda_t,\, K_t^k,\, E_t,\, P_t^k,\, r_t^k,\, q_t^k,\, Y_t^k,\, Y_t,\, W_t^k,\, M_t,\, I_t^k\}$ and, by reduction, three exogenous processes $\{A_t^k,\, a_t\}$ for $k \in \{d, g\}$.
Note that all conditions are derived under the assumption that $\mu = -a_t$, namely agents have worst-case or pessimistic beliefs (denoted by $E_t^{-a}[.]$) about the evolution of the exogenous process A_t^g.

Households

$$C_t^{-\gamma} = \lambda_t \tag{A-1}$$

$$(H_t^k)^{\sigma_H} = \lambda_t W_t^k \tag{A-2}$$

$$\mathbb{E}_t^{-a}\, r_{t+1}^k = \frac{1}{\beta} q_t^k\, \mathbb{E}_t^{-a}\, \frac{\lambda_t}{\lambda_{t+1}} - (1-\delta)\, \mathbb{E}_t^{-a}\, q_{t+1}^k + \mathbb{E}_t^{-a} \left[\frac{\phi_I}{2} \left(\frac{I_{t+1}^k}{K_{t+1}^k} - \delta \right)^2 - \phi_I \left(\frac{I_{t+1}^k}{K_{t+1}^k} - \delta \right) \frac{I_{t+1}^k}{K_{t+1}^k} \right] \tag{A-3}$$

$$q_t^k = \phi_I \left(\frac{I_t^k}{K_t^k} - \delta \right) + 1 \tag{A-4}$$

$$K_{t+1}^k = I_t^k + (1-\delta) K_t^k - \frac{\phi_I}{2} \left(\frac{I_t^k}{K_t^k} - \delta \right)^2 K_t^k \tag{A-5}$$

Dirty sector

$$Y_t^d = [1 - D_t(M_t)] A_t^d (K_t^d)^{\alpha} (H_t^d)^{1-\alpha} \tag{A-6}$$

$$r_t^d = \alpha_1 \frac{P_t^d Y_t^d}{K_t^d} \tag{A-7}$$

$$W_t^d = (1-\alpha) \frac{P_t^d Y_t^d}{H_t^d} - \phi_H \left(\frac{H_t^d}{H_{t-1}^d} - 1 \right) \frac{1}{H_{t-1}^d} + \phi_H \beta\, \mathbb{E}_t^{-a}\, \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{H_{t+1}^d}{H_t^d} - 1 \right) \frac{H_{t+1}^d}{(H_t^d)^2} \tag{A-8}$$

where adjustment costs on labor of the form $\frac{\phi_H}{2} \left(\frac{H_t^d}{H_{t-1}^d} - 1 \right)^2$ are assumed.

Green sector

$$Y_t^g = [1 - D_t(M_t)] A_t^g (K_t^g)^{\alpha} (H_t^g)^{1-\alpha} \tag{A-9}$$

$$r_t^g = \alpha_1 \frac{P_t^g Y_t^g}{K_t^g} \tag{A-10}$$

$$W_t^g = (1-\alpha)\frac{P_t^g Y_t^g}{H_t^g} - \phi_H \left(\frac{H_t^g}{H_{t-1}^g} - 1\right)\frac{1}{H_{t-1}^g} + \phi_H \beta\, \mathbb{E}_t^{-a} \frac{\lambda_{t+1}}{\lambda_t}\left(\frac{H_{t+1}^g}{H_t^g} - 1\right)\frac{H_{t+1}^g}{(H_t^g)^2} \tag{A-11}$$

where adjustment costs on labor of the form $\frac{\phi_H}{2}\left(\frac{H_t^g}{H_{t-1}^g} - 1\right)^2$ are assumed.

Final-good producers

$$Y_t = \left(\nu^{\frac{1}{\rho_Y}} (Y_t^d)^{\frac{\rho_Y - 1}{\rho_Y}} + (1-\nu)^{\frac{1}{\rho_Y}} (Y_t^g)^{\frac{\rho_Y - 1}{\rho_Y}}\right)^{\frac{\rho_Y}{\rho_Y - 1}} \tag{A-12}$$

$$P_t^d = \nu^{\frac{1}{\rho_Y}} \left(\frac{Y_t^d}{Y_t}\right)^{-\frac{1}{\rho_Y}} \tag{A-13}$$

$$P_t^g = (1-\nu)^{\frac{1}{\rho_Y}} \left(\frac{Y_t^g}{Y_t}\right)^{-\frac{1}{\rho_Y}} \tag{A-14}$$

Emissions, pollution and damage

$$E_t = \chi (Y_t^d)^{\epsilon}, \tag{A-15}$$

$$1 - D_t(M_t) \equiv \exp\left[-\xi\left(M_t - \bar{M}\right)\right]$$

$$M_t - \bar{M} = \sum_{s=0}^{t+T} (1-\delta_M)^s E_{t-s} \tag{A-16}$$

Market clearing

$$Y_t = C_t + I_t^d + I_t^g + \frac{\phi_H}{2}\left(\frac{H_t^d}{H_{t-1}^d} - 1\right)^2 + \frac{\phi_H}{2}\left(\frac{H_t^g}{H_{t-1}^g} - 1\right)^2 + \frac{\phi_I}{2}\left(\frac{I_t^d}{K_t^d} - \delta\right)^2 K_t^d + \frac{\phi_I}{2}\left(\frac{I_t^g}{K_t^g} - \delta\right)^2 K_t^g \tag{A-17}$$

Exogenous processes

$$\log A_{t+1}^d = \rho_{A^d} \log A_t^d + \varepsilon_{A^d, t+1} \tag{A-18}$$

$$\log A_{t+1}^g = \rho_{A^g} \log A_t^g - a_t + \varepsilon_{A^g, t+1} \tag{A-19}$$

$$\log a_{t+1} = \rho_a \log a_t + (1-\rho_a) \log \bar{a} + \varepsilon_{a, t+1} \tag{A-20}$$

Appendix B: Reduced model

Appendix B.1: Analytical derivation

The planner is faced with the following problem:

$$\max_{C_t, Y_t, H_t^d, H_t^g} U_t = \frac{1}{1-\gamma} C_t^{(1-\gamma)} - \sigma_H H_t^d - \sigma_H H_t^g + \beta \min_{\mu_t \in [-a_t, a_t]} \mathbb{E}_t^{\mu}[U_{t+1}] \quad \text{(B-1)}$$

subject to (15), (16) and the resource constraint of the economy $Y_t = C_t$. For tractability, disutility from labor enters the period utility linearly. The problem can be reduced to:

$$\max_{H_t^d, H_t^g} U_t = \frac{1}{1-\gamma} \left[(A_t^d H_{t-1}^d)^{\nu} (A_t^g H_{t-1}^g)^{(1-\nu)} \right]^{(1-\gamma)} - \sigma_H H_t^d - \sigma_H H_t^g + \beta \, \mathbb{E}_t^{-a}[U_{t+1}] \quad \text{(B-2)}$$

First order conditions from the problems are:

$$\text{FOC } H_t^g : \beta \, \mathbb{E}_t^{-a} \left\{ \left[(A_{t+1}^d H_t^d)^{\nu} (A_{t+1}^g H_t^g)^{(1-\nu)} \right]^{(-\gamma)} (1-\nu)(A_{t+1}^d H_t^d)^{\nu} (A_{t+1}^g)^{(1-\nu)} (H_t^g)^{(-\nu)} \right\} = \sigma_H \quad \text{(B-3)}$$

$$\text{FOC } H_t^d : \beta \, \mathbb{E}_t^{-a} \left\{ \left[(A_{t+1}^d H_t^d)^{\nu} (A_{t+1}^g H_t^g)^{(1-\nu)} \right]^{(-\gamma)} \nu (A_{t+1}^d)^{\nu} (H_t^d)^{(\nu-1)} (A_{t+1}^g H_t^g)^{(1-\nu)} \right\} = \sigma_H \quad \text{(B-4)}$$

The ratio of (B-4) and (B-3) gives out the relation:

$$H_t^d = \frac{\nu}{1-\nu} H_t^g \quad \text{(B-5)}$$

Plugging (B-5) into (B-3) and rearranging:

$$(H_t^g)^{-\gamma} \, \mathbb{E}_t^{-a} \left\{ (A_{t+1}^d)^{\nu(1-\gamma)} (A_{t+1}^g)^{(1-\nu)(1-\gamma)} \right\} = \frac{\sigma_H (1-\nu)^{\nu(1-\gamma)-1}}{\beta \nu^{\nu(1-\gamma)}} \quad \text{(B-6)}$$

Similarly, it is possible to retrieve the condition for H_t^d:

$$(H_t^d)^{-\gamma} \, \mathbb{E}_t^{-a} \left\{ (A_{t+1}^d)^{\nu(1-\gamma)} (A_{t+1}^g)^{(1-\nu)(1-\gamma)} \right\} = \frac{\sigma_H \nu^{(1-\nu)(1-\gamma)-1}}{\beta (1-\nu)^{(1-\nu)(1-\gamma)}} \quad \text{(B-7)}$$

Using normality of the shocks, under independence of the two exogenous processes and $\mu_t = -a_t$, the closed-form solution for hours worked in the two sectors can be rewritten in logarithmic terms as:

$$h_t^g = \frac{1}{\gamma} \left[(1-\nu)(1-\gamma)(-a_t) - \bar{hg} + \Sigma \right] \quad \text{(B-8)}$$

$$h_t^d = \frac{1}{\gamma} \left[(1-\nu)(1-\gamma)(-a_t) - \bar{hd} + \Sigma \right] \quad \text{(B-9)}$$

where $\bar{hg} = log\left(\frac{\sigma_H}{\beta}\right) + [\nu(1-\gamma) - 1]log(1-\nu) - \nu(1-\gamma)log(\nu)$ and $\bar{hd} = log\left(\frac{\sigma_H}{\beta}\right) + [(1-\nu)(1-\gamma) - 1]log(\nu) - (1-\nu)(1-\gamma)log(1-\nu)$, while Σ is a compound term depending on the variances of both processes equal to $[\nu^2(1-\gamma)^2]\sigma_{A^d}^2/2 + [(1-\nu)^2(1-\gamma)^2]\sigma_{A^g}^2/2$.

Appendix B.2: Optimal tax and ambiguity

The planner is faced with the same problem as in (B-1), but now (15) is changed to (20) and there is an additional constraint for emissions $E_t = (A_t^d H_{t-1}^d)^{\epsilon}$, with associated multiplier λ_t. First order conditions from the problems are:

$$\text{FOC } H_t^g: \ (1-\nu)\beta\,\mathbb{E}_t^{-a}\left\{\left[e^{(-\xi E_{t+1})}\right](A_{t+1}^d H_t^d)^{\nu}(A_{t+1}^g H_t^g)^{(1-\nu)}\right\}^{(1-\gamma)}(H_t^g)^{(-1)} = \sigma_H \tag{B-10}$$

$$\begin{aligned}\text{FOC } H_t^d: \nu\beta\,\mathbb{E}_t^{-a}\left\{\left[e^{(-\xi E_{t+1})}\right](A_{t+1}^d H_t^d)^{\nu}(A_{t+1}^g H_t^g)^{(1-\nu)}\right\}^{(1-\gamma)}(H_t^d)^{-1} \\ +\,\beta\,\mathbb{E}_t^{-a}\left\{\lambda_{t+1}\epsilon(A_{t+1}^d)^{\epsilon}(H_t^d)^{(\epsilon-1)}\right\} = \sigma_H\end{aligned} \tag{B-11}$$

$$\text{FOC } E_t: \ \xi\left\{\left[e^{(-\xi E_t)}\right](A_t^d H_{t-1}^d)^{\nu}(A_t^g H_{t-1}^g)^{(1-\nu)}\right\}^{(1-\gamma)} = \lambda_t \tag{B-12}$$

The difference between (B-4) and (B-11) shapes the optimal tax that the planner can impose to reinstate the first best:

$$\tau_t^d \propto \beta\xi\epsilon\,\mathbb{E}_t^{-a_t}\left\{\left\{\left[e^{(-\xi E_{t+1})}\right](A_{t+1}^d H_t^d)^{\nu}(A_{t+1}^g H_t^g)^{(1-\nu)}\right\}^{(1-\gamma)}(A_{t+1}^d)^{\epsilon}(H_t^d)^{(\epsilon-1)}\right\} \tag{B-13}$$

Considering the problem in steady state, the optimal tax boils down to:

$$\tau_{amb}^d \propto \beta\xi\epsilon\left\{\left\{\left[e^{(-\xi E)}\right](H^d)^{\nu}(e^{-a}H^g)^{(1-\nu)}\right\}^{(1-\gamma)}(H^d)^{(\epsilon-1)}\right\} \tag{B-14}$$

In a no-ambiguity scenario, the optimal instrument would instead be proportional to:

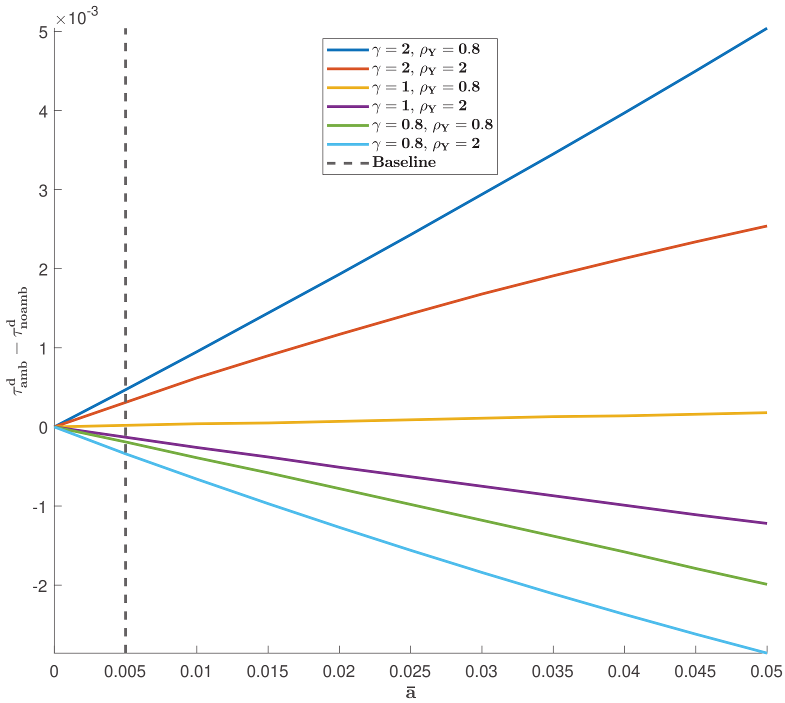
$$\tau_{noamb}^d \propto \beta\xi\epsilon\left\{\left\{\left[e^{(-\xi E)}\right](H^d)^{\nu}(H^g)^{(1-\nu)}\right\}^{(1-\gamma)}(H^d)^{(\epsilon-1)}\right\} \tag{B-15}$$

Comparing (B-14) and (B-15):

$$\tau_{amb}^d = e^{-\bar{a}(1-\nu)(1-\gamma)}\tau_{noamb}^d \tag{B-16}$$

Moving from the tractable case analyzed above to one where the aggregator is of the CES kind, Figure B-1 reports additional quantitative sensitivity checks to test the response of the instrument to changes in relevant parameters. The direction of the effect of an increase in the steady-state value of ambiguity (in the worst case) on the optimal constant environmental tax is driven mainly by risk aversion γ. When the CES case is considered, only the sensitivity of the response is affected. When the inputs from the two sectors are less and less substitutes (or even complements), a decrease in confidence induces a bigger increase in total production value. In fact, while agents are pushed to work relatively less in the supposedly less productive sector in the baseline calibration, this substitution effect is weakened when $\rho_Y < 1$.

Figure B-1: Deviation from no-ambiguity optimal environmental tax



Note: The figure shows the p.p. deviation from the no-ambiguity optimal environmental tax as steady-state ambiguity $\bar{a}$ increases for different values of risk aversion γ and elasticity of substitution ρ_Y.

Appendix B.3: Details on welfare cost

The welfare cost of ambiguity is given by ς such that:

$$\frac{u^0(C(1-\varsigma), H^d, H^g)}{1-\beta} = \underline{U}^{-a} \tag{B-17}$$

where

$$u^0 = \frac{C^{1-\gamma}}{1-\gamma} - \sum_{k\in\{d,g\}} \frac{(H^k)^{1+\sigma_H}}{1+\sigma_H} \tag{B-18}$$

and

$$\underline{U}^{-a} = \frac{\left(\frac{\underline{C}^{1-\gamma}}{1-\gamma} - \sum_{k\in\{d,g\}} \frac{(\underline{H}^k)^{1+\sigma_H}}{1+\sigma_H}\right)}{1-\beta} \tag{B-19}$$

where $\underline{C}$, $\underline{H}^d$ and $\underline{H}^g$ are the steady-state values of consumption and hours under ambiguity. ς is then computed as:

$$\varsigma = 1 - \left[\frac{\underline{U}^{-a} + \frac{\sigma_H}{1-\beta}(H^d + H^g)}{\frac{C^{1-\gamma}}{(1-\gamma)(1-\beta)}}\right]^{\frac{1}{1-\gamma}} \tag{B-20}$$

Appendix C: Full model

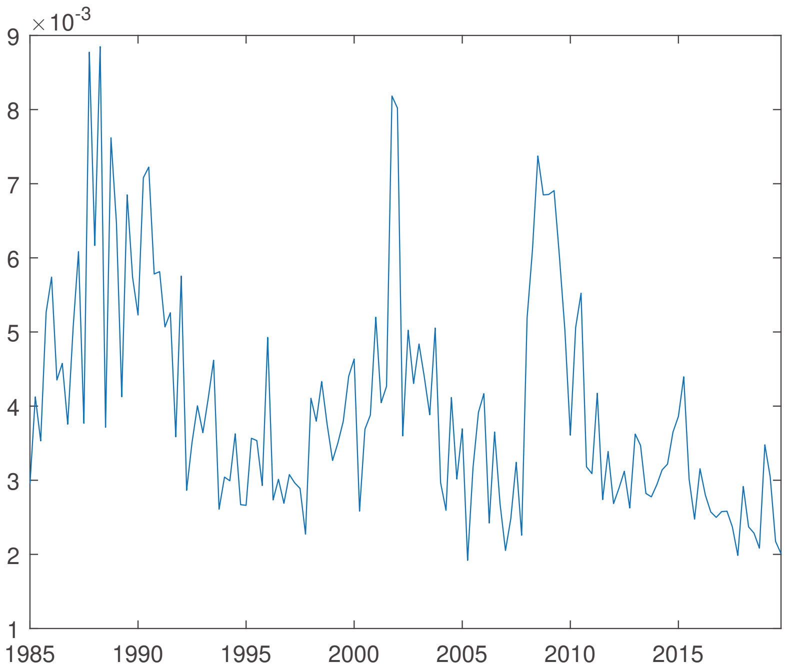
Appendix C.1: Calibration details

Figure C-1 shows the measure of dispersion for the quarterly forecasts of US real GDP taken as reference for calibration from SPF (mean is 0.004 and standard deviation 0.0015). The model counterpart of this measure is the range of one quarter ahead forecasts for the log level of GDP implied by agents' belief set. Following Ilut and Schneider (2014), this range of forecasts is defined as:

$$D_t = \iota_{y,A^g} 2|a_t|$$

where ι_{y,A^g} is the response of log GDP to green technology innovation ε_{A^g} in the linearized solution.

Figure C-1: Cross-sectional dispersion for quarterly forecasts for US real GDP



Note: Difference between the 75th percentile and the 25th percentile of the projections for the quarterly level (in log units) of US real GDP from the Survey of Professional Forecasters (SPF).

Starting from the model variables consumption C, investments I and output Y net of adjustment costs, new variables $c_t = \log(C_t) - \log(C)$, $i_t = \log(I_t) - \log(I)$ and $y_t = \log(Y_t) - \log(Y)$ are defined. Simulated moments are then generated by the model and compared to those observed in the US data. Table C-1 reports the results.

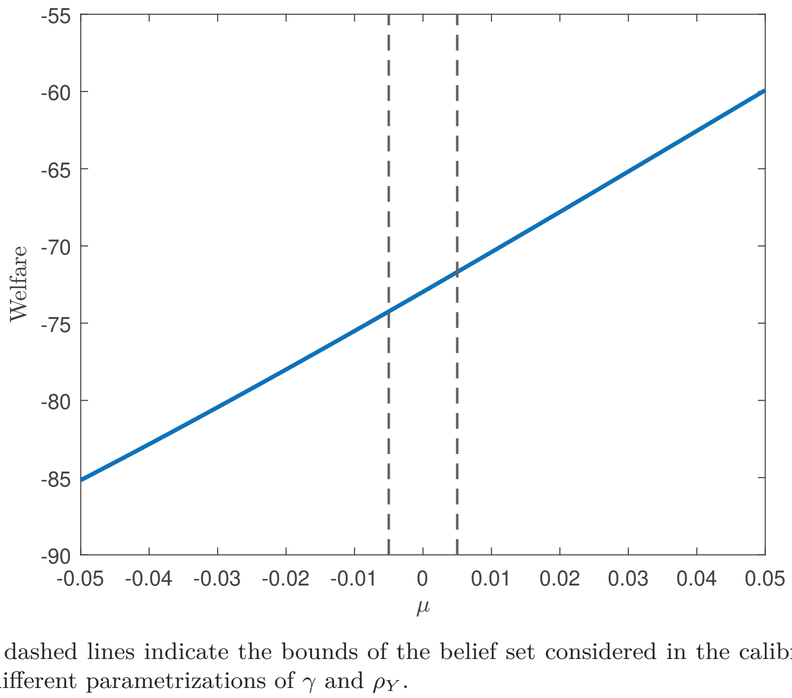
Table C-1: Model and Data - Moments

	Model	Data
Standard Deviation		
σ_y	0.012	0.010
σ_i/σ_y	3.69	4.67
Cross-Correlations		
$\rho_{i,y}$	0.95	0.89
$\rho_{c,y}$	0.84	0.92
First-Order Autocorrelation		
ρ_y	0.92	0.90
ρ_i	0.88	0.88

Note: the table reports the moments generated by the model and those of the US HP-filtered quarterly data over the period 1985Q1-2019Q4, retrieved from FRED.

Appendix C.2: Worst-case choice

Figure C-2: Steady-state welfare as a function of ambiguous component μ



Note: Vertical dashed lines indicate the bounds of the belief set considered in the calibration. Results are robust to different parametrizations of γ and ρ_Y.

Steady-state welfare is defined as:

$$\underline{U}^{-a} = \frac{\left(\frac{\underline{C}^{1-\gamma}}{1-\gamma} - \sum_{k \in \{d,g\}} \frac{(\underline{H}^k)^{1+\sigma_H}}{1+\sigma_H} \right)}{1-\beta}$$

Figure C-2 shows that the above measure is increasing in the values that the ambiguous component μ can take in any given belief interval. In other words, steady-state welfare is minimal when μ takes the value of the lower bound of $[-a_t, a_t]$. Thus, setting $\mu_t = -a_t$ as the minimizer of the continuation utility is a valid choice.

Appendix C.3: Optimal tax: analytical derivation

The social planner chooses $C_t, I_t^k, Y_t^k, K_{t+1}^k, H_t^k, M_t$ for $k \in \{d, g\}$ with the objective of maximizing (12) subject to (1), (A-17) and (14) net of adjustment costs, (7), (2) and (5). Ultimately, the planner solves the centralized problem with the same preferences specification as the agents, thus the results that follow are effectively not a standard first best outcome (although ambiguity does not appear explicitly). Note that condition (5) is rewritten for convenience in a recursive formulation that well approximates it for a time horizon that moves towards infinity:

$$M_t - \bar{M} = (1-\delta_M)\left(M_{t-1} - \bar{M}\right) + E_t + E_t^{\star}, \tag{C-1}$$

A selection of useful first order conditions from the problem read as:

$$\text{FOC } Y_t: \; \lambda_t^1 = -\lambda_t^2$$

$$\text{FOC } C_t: \; \frac{1}{C_t} = -\lambda_t^2$$

$$\text{FOC } Y_t^d: \; \lambda_t^1 \nu^{\frac{1}{\rho_Y}} \left(\frac{Y_t^d}{Y_t}\right)^{-\frac{1}{\rho_Y}} - \lambda_t^6 + \lambda_t^7 \chi \epsilon (Y_t^d)^{\epsilon-1} = 0$$

$$\text{FOC } M_t: \; \lambda_t^7 = \xi(\lambda_t^5 Y_t^g + \lambda_t^6 Y_t^d) + \beta(1-\delta_M)\,\mathbb{E}_t^{-a}\,\lambda_{t+1}^7$$

where λ_t^1 is the multiplier associated with (1), λ_t^2 with (A-17), λ_t^5 with (7), λ_t^6 with (2) and λ_t^7 with (C-1). Notice that, since $\exp(-\xi(M_t - \bar{M})) \equiv 1 - D_t(M_t)$, it's also true that differentiating with respect to M_t gives $-\xi \exp(-\xi(M_t - \bar{M})) \equiv -D_t'(M_t)$. To capture the absolute value of the marginal damage, the derivative is coherently rendered positive. The optimality condition with respect to dirty production implies:

$$\frac{\lambda_t^6}{\lambda_t^1} = \nu^{\frac{1}{\rho_Y}} \left(\frac{Y_t^d}{Y_t}\right)^{-\frac{1}{\rho_Y}} + \frac{\lambda_t^7}{\lambda_t^1} \epsilon (Y_t^d)^{\epsilon-1}$$

The optimality condition with respect to pollution stock implies:

$$\frac{\lambda_t^7}{\lambda_t^1} = \xi\left(\frac{\lambda_t^5}{\lambda_t^1} Y_t^g + \frac{\lambda_t^6}{\lambda_t^1} Y_t^d\right) + \beta(1-\delta_M)\,\mathbb{E}_t^{-a}\left[\frac{\lambda_{t+1}^7}{\lambda_{t+1}^1}\frac{C_t}{C_{t+1}}\right]$$

Define $\frac{\lambda_t^5}{\lambda_t^1}$ as $\tilde{P}_t^g$ and $\frac{\lambda_t^6}{\lambda_t^1}$ as $\tilde{P}_t^d$, given that the ratios can be interpreted as the shadow price of input g, d at time t, and $\frac{\lambda_t^7}{\lambda_t^1}$ as $\tilde{P}_t^M$ to identify the discounted marginal damage from emissions on total production value. Then

$$\tilde{P}_t^d = \nu^{\frac{1}{\rho_Y}} \left(\frac{Y_t^d}{Y_t}\right)^{-\frac{1}{\rho_Y}} + \tilde{P}_t^M \chi\epsilon (Y_t^d)^{\epsilon-1} \tag{C-2}$$

$$\tilde{P}_t^M = \xi \left(\tilde{P}_t^g Y_t^g + \tilde{P}_t^d Y_t^d\right) + \beta(1-\delta_M)\, \mathbb{E}_t^{-a} \left[\tilde{P}_{t+1}^M \frac{C_t}{C_{t+1}}\right] \tag{C-3}$$

Comparing condition (A-13) from the decentralized equilibrium, the first equation shows how the social planner introduces a wedge between the marginal product of dirty input and its price, which is equal to the environmental cost of an additional unit of the dirty input. The second equation further specifies that the marginal cost of having an additional unit of pollution today is equal to the marginal damage on productivity plus the future discounted cost of an additional unit of pollution in the next period, net of depreciation.

This wedge is also equivalent to an *ad valorem* tax $\tau_t^{*,d} = \frac{\tilde{P}_t^M \chi\epsilon (Y_t^d)^{\epsilon-1}}{\tilde{P}_t^d}$, which in steady state reads as $\tau^{*,d} = \frac{\tilde{P}^M \chi\epsilon (Y^d)^{\epsilon-1}}{\tilde{P}^d}$ where

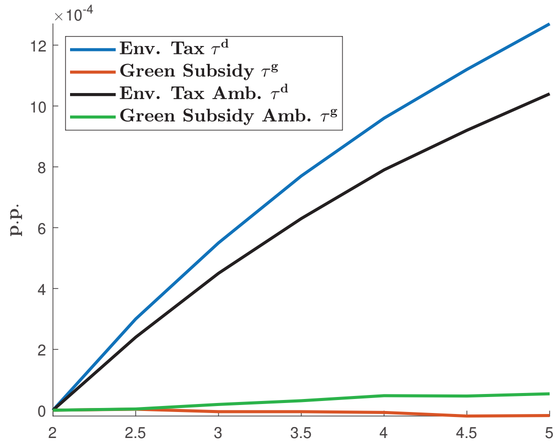
$$\tilde{P}^d = \nu^{\frac{1}{\rho_Y}} \left(\frac{Y^d}{Y}\right)^{-\frac{1}{\rho_Y}} + \tilde{P}^M \chi\epsilon (Y^d)^{\epsilon-1}$$

$$\tilde{P}^M = \frac{\xi \left(\tilde{P}^g Y^g + \tilde{P}^d Y^d\right)}{1-\beta(1-\delta_M)}$$

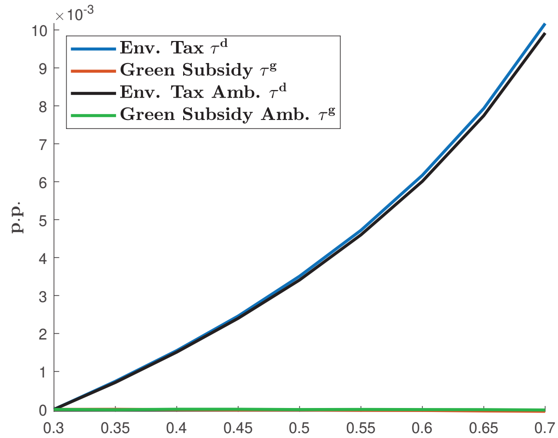
The optimal tax, together with lump-sum rebates to households, internalizes the externality of dirty production, thus implementing the ”efficient” allocation. However, notice again that this is not a proper first best outcome, as the planner is constrained by the same distorted preferences as the agents.

Appendix C.4: Policy mix: sensitivity

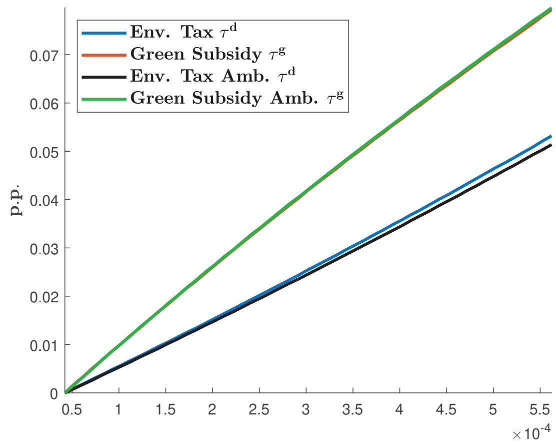
Figure C-3: Sensitivity exercises



(a) ρ_Y



(b) $1 - \nu$

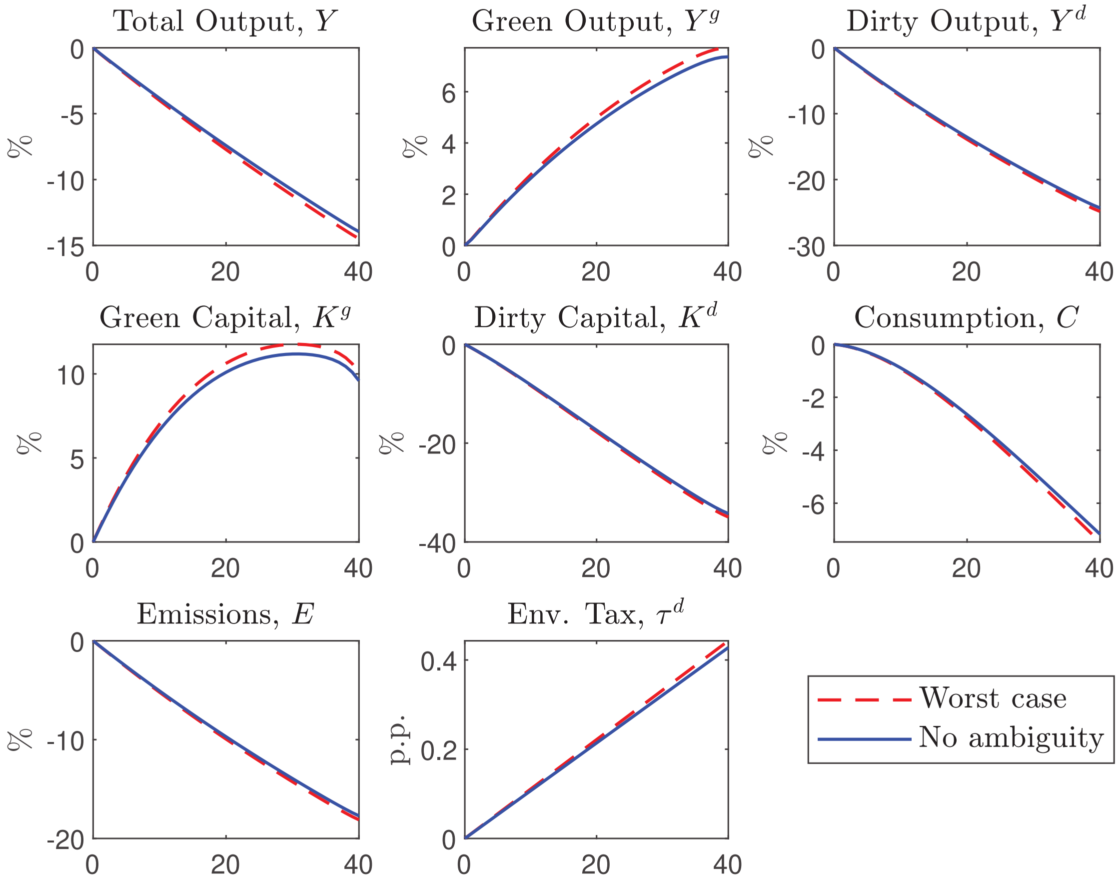


(c) ξ

Note: The three figures show the p.p. deviation from respective (zero-ambiguity and ambiguity case) optimal environmental tax and implied subsidy of the policy mix as relevant parameters change values. (a) shows the case for an increase in the elasticity of substitution ρ_Y; (b) for the share of green production $1 - \nu$; (c) for the sensitivity to damages ξ (which also implies higher damages). The black and green lines show the same exercise peformed under baseline ambiguity $\bar{a} = 0.005$.

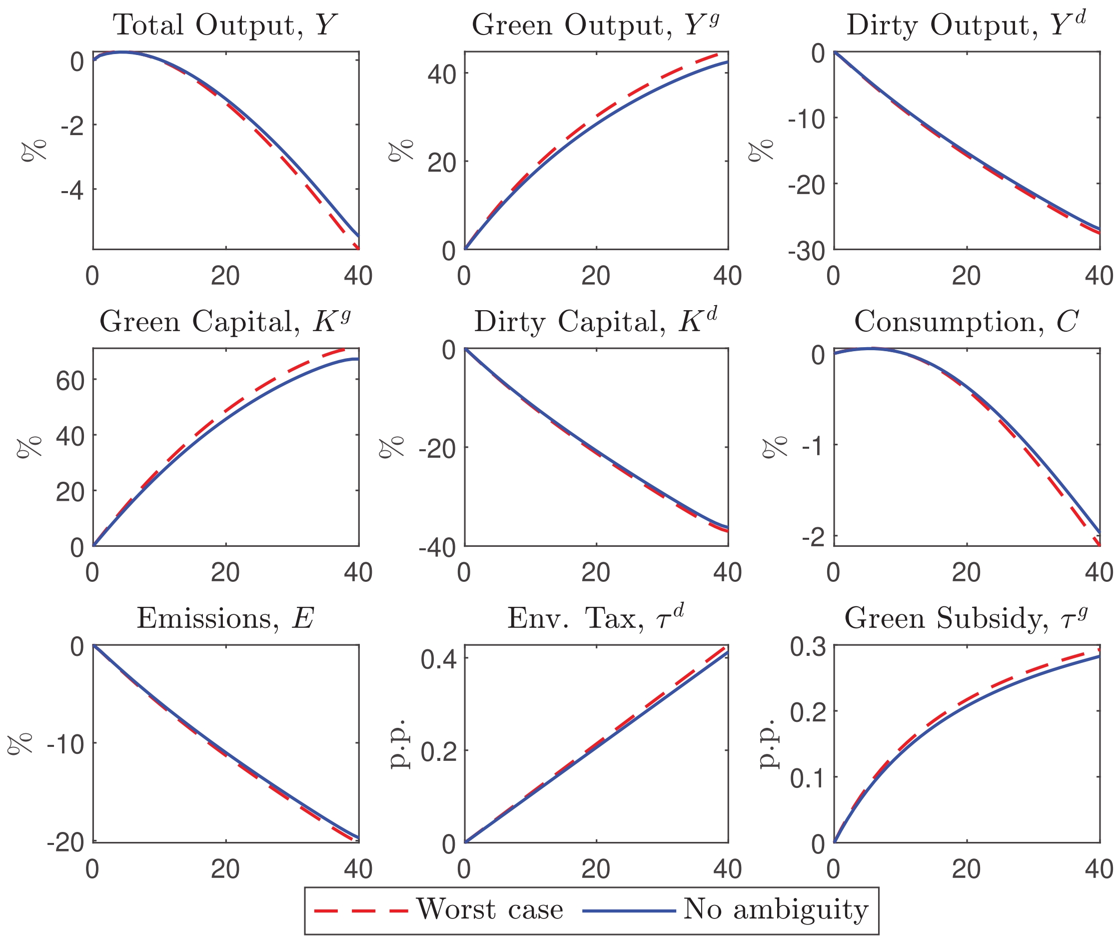
Appendix C.5: Transition exercise

Figure C-4: Transition under worst-case v. no-ambiguity



Note: Results are reported as % and p.p. deviations from the respective initial steady states. The transition under no ambiguity is driven by a environmental tax that increases linearly over 40 quarter to reach a final steady state implying around a 20% reduction in emissions. Under the worst-case, the path of the tax is adjusted to reach the same level of emissions as in the no-ambiguity scenario.

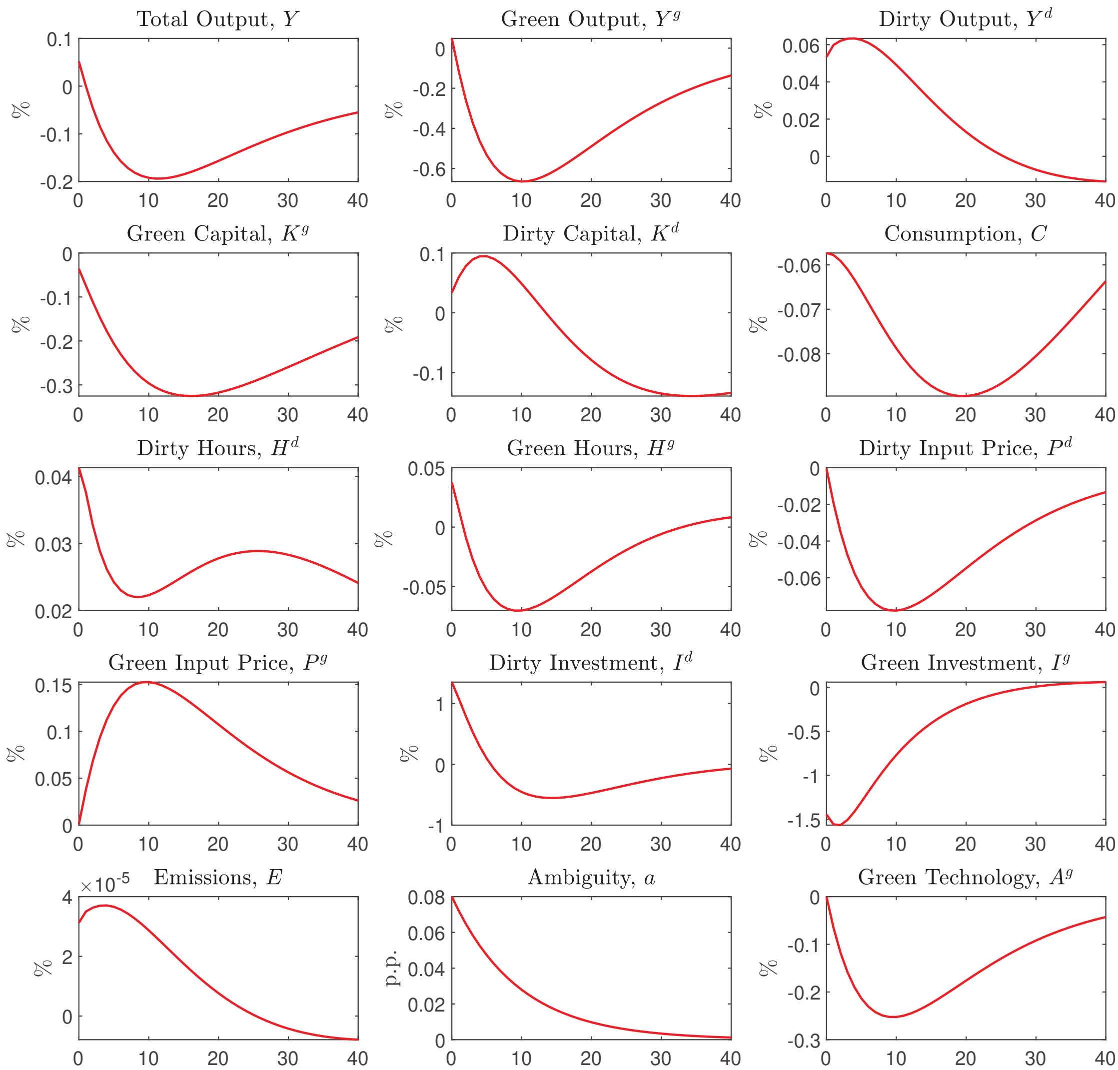
Figure C-5: Transition under worst-case v. no-ambiguity: optimal mix



Note: Results are reported as % and p.p. deviations from respective initial steady states. The transition under no ambiguity is driven by a environmental tax that increases linearly over 40 quarter to reach a final steady state implying around a 20% reduction in emissions. Under the worst-case, the path of the tax is adjusted to reach the same level of emissions as in the no-ambiguity scenario.

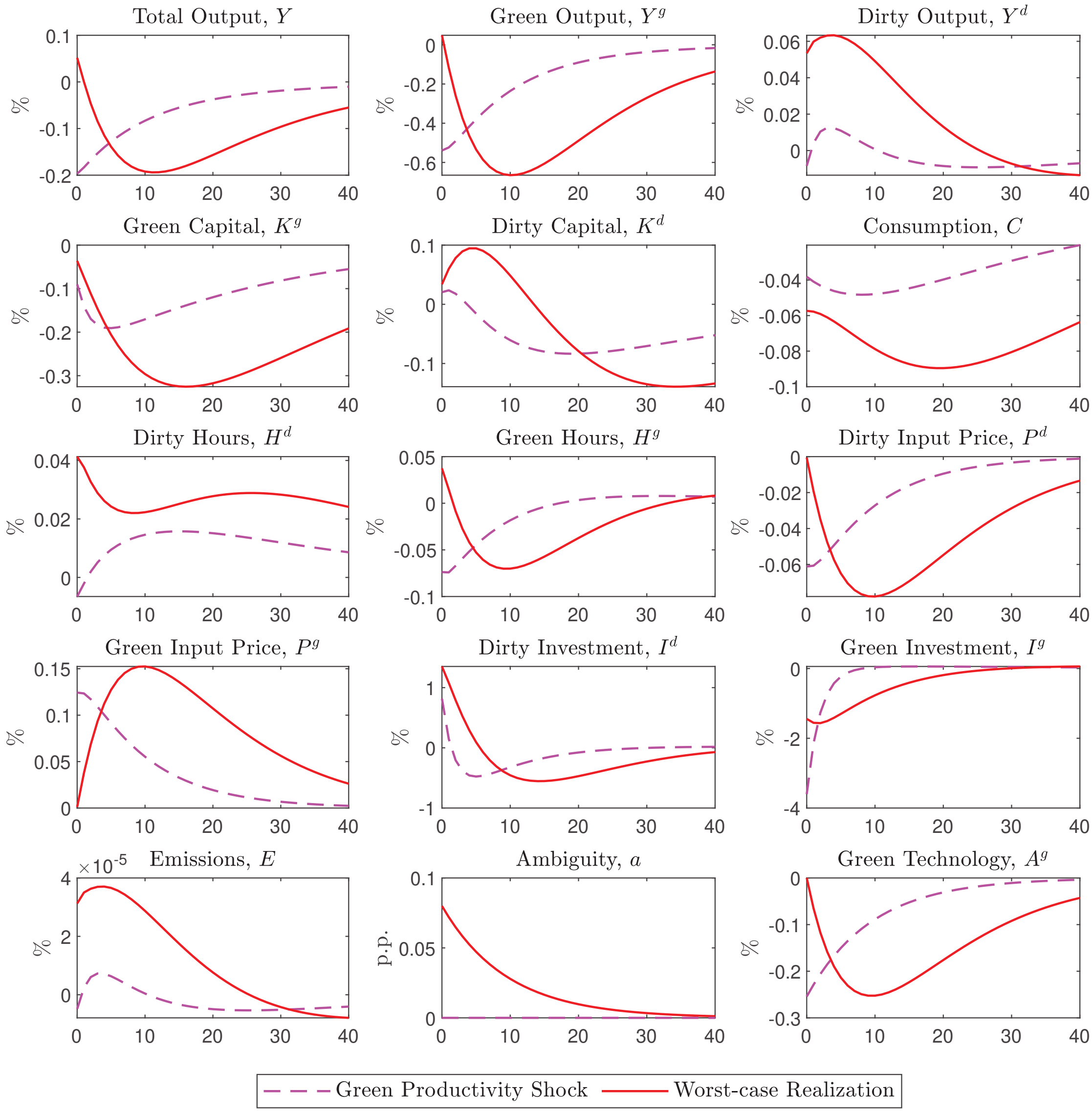
Appendix C.6: Dynamic analysis

Figure C-6: Dynamic response to a positive ambiguity shock under worst-case realization



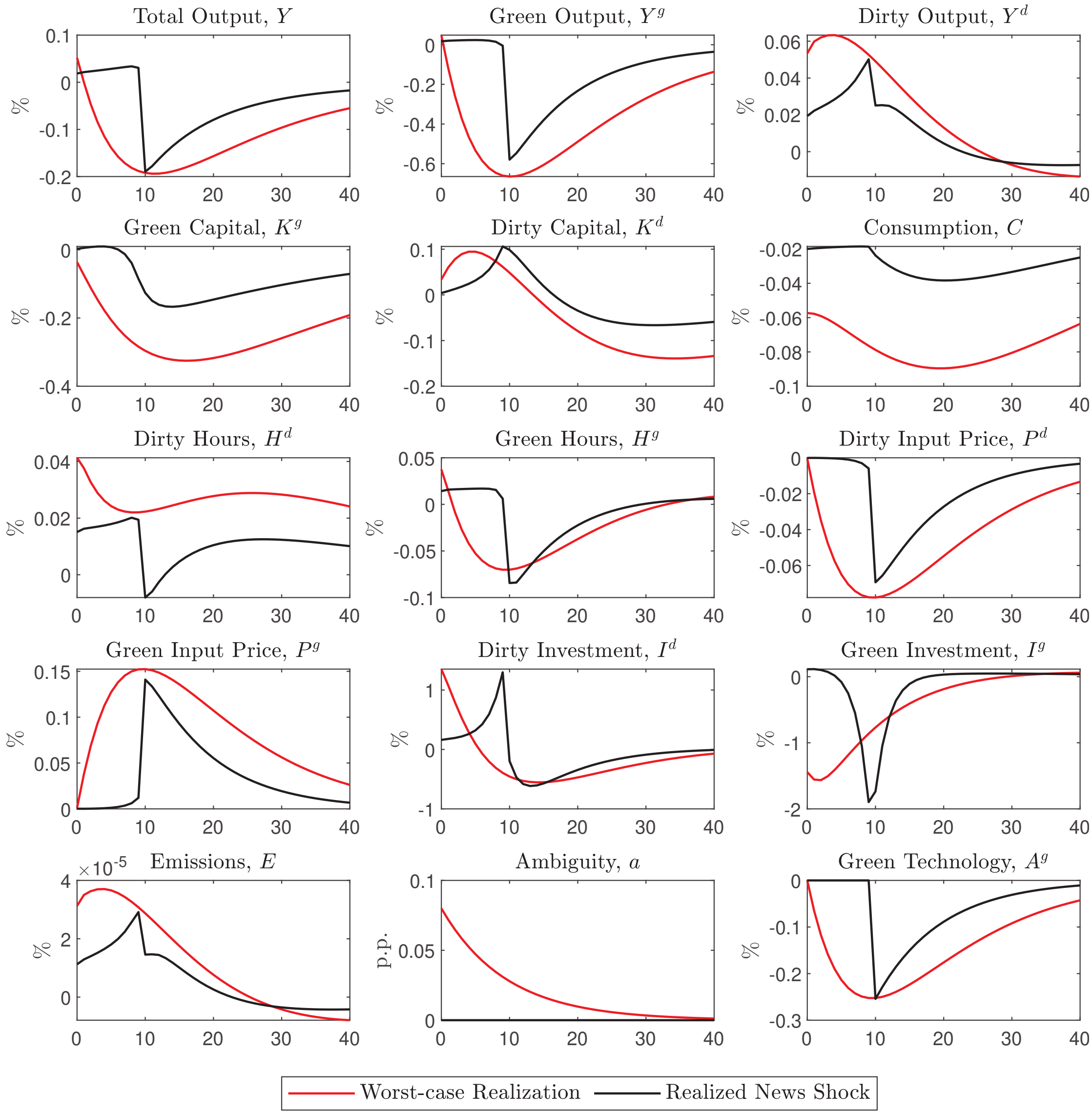
Note: Results are reported over a 40-quarter time horizon as percentage deviations from the no-ambiguity steady state.

Figure C-7: Dynamic response to worst-case realization of positive ambiguity shock v. negative green productivity shock



Note: Results are reported over a 40-quarter time horizon as percentage deviations from the no-ambiguity steady state.

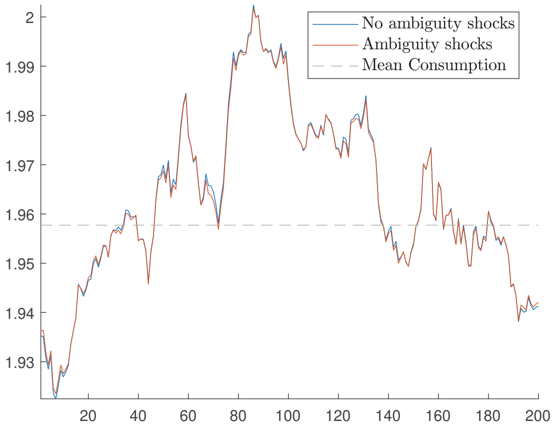
Figure C-8: Dynamic response to worst-case realization of positive ambiguity shock v. realized news shock



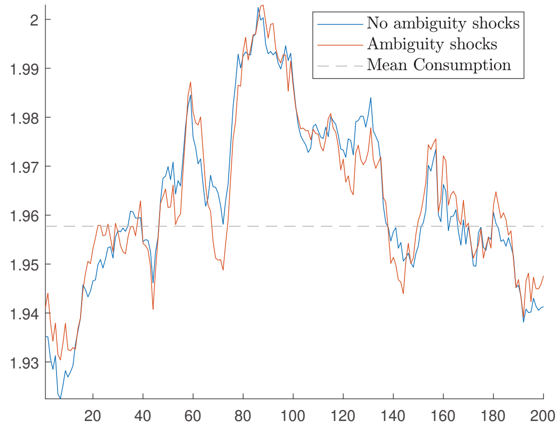
Note: Results are reported over a 40-quarter time horizon as percentage deviations from the no-ambiguity steady state.

Appendix C.7: Optimal policies: simulations

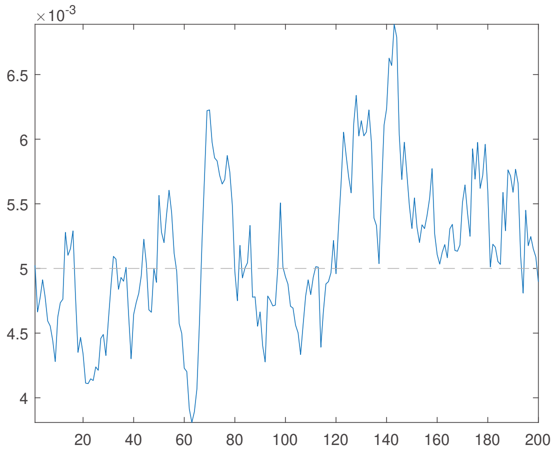
Figure C-9: Simulations



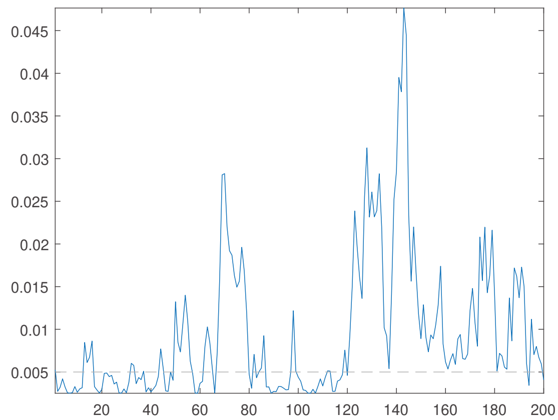
(a) Consumption, $\sigma_a = 0.05$



(b) Consumption, $\sigma_a = 0.5$



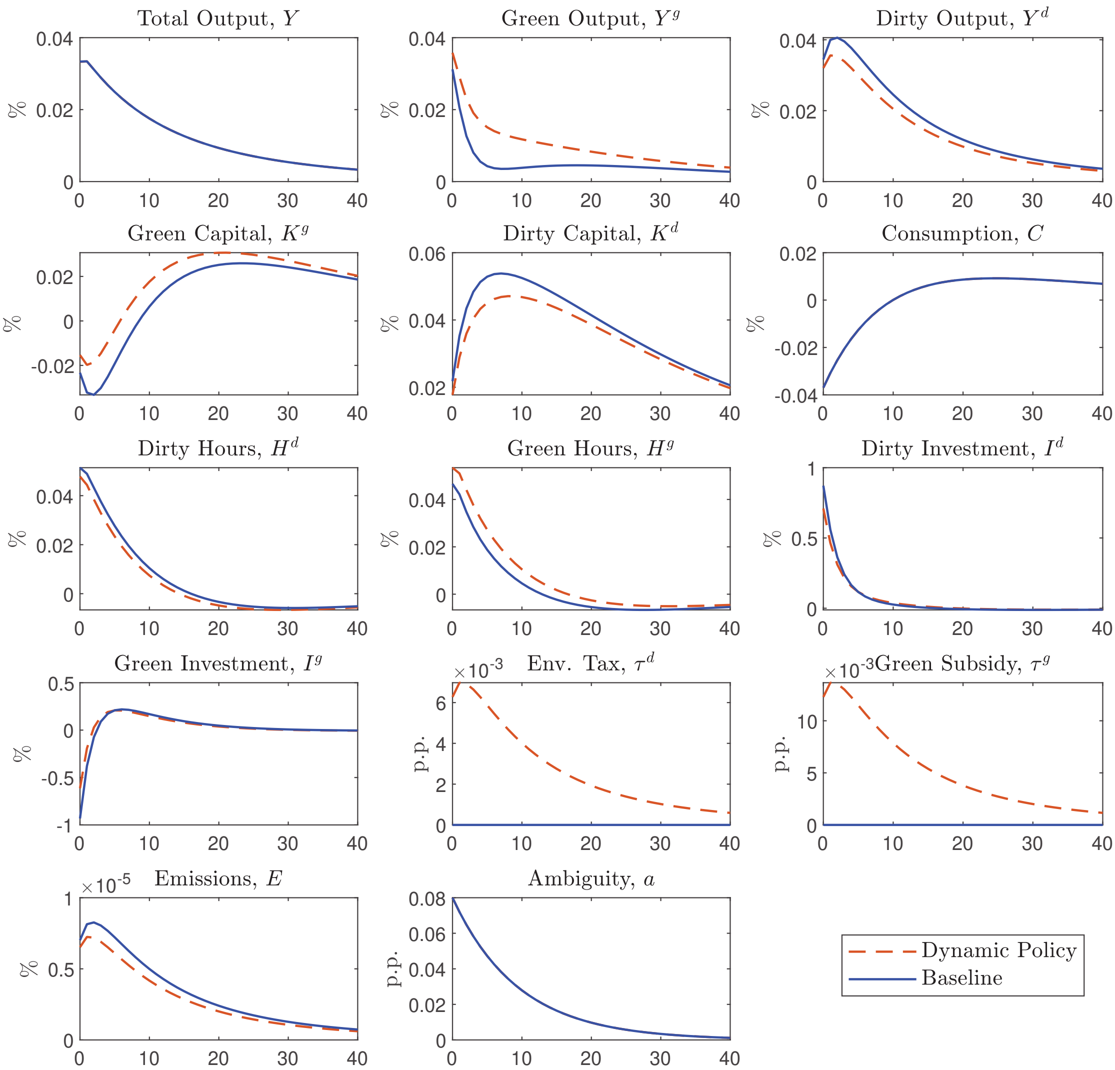
(c) Ambiguity, $\sigma_a = 0.05$



(d) Ambiguity, $\sigma_a = 0.5$

Note: The figures report simulations for consumption and variations in confidence under the baseline calibration (a) and (c) and under increased standard deviation for the ambiguity shocks (b) and (d). Dashed line in (c) and (d) represents steady-state ambiguity.

Figure C-10: Baseline v. dynamic policy



Note: Results are reported over a 40-quarter time horizon as percentage deviations from the no-ambiguity steady state.

Appendix D: Solution method

This appendix explains the analytical and computational steps for the method used to solve the model.

Appendix D.1: Analytical details

Following Ilut and Schneider (2014), the first step in solving the model is to guess a worst-case belief, which entails that agents behave as if innovations to productivity are low, namely $\mu = -a_t$, at all histories. Following the empirical rationale for this kind of belief provided in Appendix C.2, the steady-state value for A^g is equal to $\exp\left(\frac{-a_t}{1-\rho_{A^g}}\right)$. Agents act as if the economy converged in the long run to the worst-case state computed in this fashion, however agents' pessimistic expectations are not fulfilled by the realization of the exogenous process. As a consequence, the non-distorted (ergodic) steady state will generally differ from its worst-case counterpart. The exogenous laws of motion can be summarized as follows:

$$s_t^0 := \begin{pmatrix} \log A_t^d \\ \log A_t^g \\ \log a_t \end{pmatrix} = \begin{pmatrix} \rho_{A^d} & 0 & 0 \\ 0 & \rho_{A^g} & -1 \\ 0 & 0 & \rho_a \end{pmatrix} \begin{pmatrix} \log A_{t-1}^d \\ \log A_{t-1}^g \\ \log a_{t-1} \end{pmatrix} + \underbrace{\begin{pmatrix} \varepsilon_{A^d,t} \\ \varepsilon_{A^g,t} \\ \varepsilon_{a,t} \end{pmatrix}}_{\omega_t} \tag{D-1}$$

Using equilibrium conditions log-linearized around the worst-case steady state, the model can be written in the canonical form:

$$\Gamma_0 \hat{y}_t^0 = \Gamma_1 \hat{y}_{t-1}^0 + \Psi \omega_t + \Upsilon \eta_t \tag{D-2}$$

where $\hat{y}_t^0$ contains log-deviations for both endogenous and exogenous variables and η_t are possible expectational errors. Various approaches can be followed to solve for (D-2), like the method by Sims (2002) used in Ilut and Schneider (2014).

Once the model is solved, the law of motion of the economy approximated to the first order is:

$$\hat{y}_t^0 = T \hat{y}_{t-1}^0 + R \omega_t \tag{D-3}$$

Agents' response to ambiguity leads to equilibrium outcomes governed by the following process in logarithmic terms

$$\log A_{t+1}^g = \rho_{A^g} \log A_t^g - a_t + \varepsilon_{A^g,t+1} \tag{D-4}$$

On the other hand, the exogenous state a_t^g actually moves according to

$$\log A_{t+1}^g = \rho_{A^g} \log A_t^g + \varepsilon_{A^g,t+1} \tag{D-5}$$

There is therefore a difference between the actual steady state of $\log A^g$ and the average innovation of the green productivity shock at time t from the perspective of the agent's worst case beliefs at $t-1$, which amounts to a_{t-1} as made evident by comparing (D-4) and (D-5). The corrected equilibrium law of motion is then given by

$$\hat{y}_t^0 = T \hat{y}_{t-1}^0 + R \omega_t + R[0 \;\; a_{t-1} \;\; 0]' \tag{D-6}$$

Appendix D.2: Dynare implementation

This subsection reports the implementation of the correction by Ilut and Schneider (2014) in the Dynare software. This allows to avoid analytical approximations of equilibrium conditions and exploit all the functionalities of the software that directly operates from the corrected policy function. This approach also offers the possibility to better handle medium-scale models.

Following the solution framework by Schmitt-Grohe and Uribe (2004) and applying notation consistent with Dynare, the correction for the first-order approximation is given by:

$$\hat{y}_t^0 = ghx\;\hat{y}_{t-1}^0 + ghu\;\omega_t + ghu\;[0\;\; a_{t-1}\;\; 0]' \tag{D-7}$$

An expansion of the correction to the second order entails an additional term to get rid of the second order distortion of ambiguity on green productivity and reads:

$$\begin{aligned}\hat{y}_t^0 &= ghs2 + ghx\;\hat{y}_{t-1}^0 + ghu\;\omega_t + ghu\;[0\;\; a_{t-1}\;\; 0]' + ghxx\;(\hat{y}_{t-1}^0 \otimes \hat{y}_{t-1}^0)\\ &\quad + \frac{1}{2}ghuu\;(\omega_t \otimes \omega_t) - \frac{1}{2}ghuu\;[\mathbf{0}_{4x1}\;\; a_{t-1}\;\; \mathbf{0}_{4x1}]' + ghxu\;(\hat{y}_{t-1}^0 \otimes \omega_t)\end{aligned} \tag{D-8}$$

Note: whenever second-order effects are less relevant, it is usually enough to apply the first-order correction even when solving at the second order. This can also help avoid potential additional instability induced by the second-order correction.

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